MATH 335: Numerical Analysis

Problem Set 18, Final version

Due Date: Mon., Apr. 27, 2009

Read Sections 11.1 (optionally 11.2), 11.3–11.4 from Kharab & Guenther, along with your in-class notes.

⋆38 (a) We have discussed two (of the many) 2^{nd}-order Runge-Kutta methods: Heun's method and the modified Euler method. (Algorithms for both are provided in boxes on the handout.) You may download my algorithm for Heun's method, which is different from the one provided by Kharab & Guenther (the one *they* call "modified Euler") in that it makes no attempt to print results to the screen, but rather returns approximate values of the solution in a vector.

Study the differences in the algorithms for Heun and modified Euler. Then write a function which, if written in OCTAVE, should have declaration

```
function [y, t] = modEuler(f, t0, tlast, y0, N)
```

with these inputs/outputs having the same meaning as they have for my routine.

(b) Write programs to carry out Euler's algorithm and the 4th-order Runge-Kutta algorithm given in class. Both programs should have declarations which are like the one above for `modEuler()`. Note that there are routines supplied with your text for these two methods. The essential difference will be that yours *returns* values instead of printing them to the screen.

(c) Test your routines. I might suggest first trying out your Euler routine on a solved example from Section 11.1 (perhaps Example 11.3, pp. 374–357) and your 4th-order Runge-Kutta routine on one from Section 11.4 (perhaps Example 11.8, pp. 393–395, for which the 4^{th}-order method's values appear in Table 11.7). Then compare all four methods on the following IVP:

$$y' = \frac{ty - y^2}{t^2}, \qquad y(1) = 2 .$$

It is possible to solve this IVP analytically; the solution is $y(t) = 2t/(1 + 2\ln t)$. Use this fact to verify (or determine) whether the advertised global truncation error for each is realized on this problem. Do the two 2^{nd}-order methods do equally well?

11.4.3 Use the Runge-Kutta method of order 4 with $h = 0.1$ to approximate the solution of the IVP

$$y' = y - \frac{y}{t}, \quad t \in [1,2], \qquad \text{subject to} \qquad y(1) = 2 .$$

Use the `polyfit()` function along with the data points generated by the Runge-Kutta method to find the best *quadratic* function that fits this data in the least squares sense. Use the resulting function to approximate these values of the true solution y: $y(1.02)$, $y(1.67)$, and $y(1.98)$.

CP 11.4.6 To show an interesting fact about Runge-Kutta methods, we consider the following initial-value problems

$$y' = 2(t+1), \quad y(0) = 1,$$

$$y' = \frac{2y}{t+1}, \quad y(0) = 1 .$$

They both have the same solution $y = (t+1)^2$.

(a) Use Heun's method with $h = 0.1$ to approximate the solution in $[0, 1]$ of the two IVPs.

(b) Compare the approximate solutions with the actual values of y.

(c) Show that for the first equation, Heun's method gives the exact results, but not for the 2^{nd} equation, although the exact solution is the same for both equations. The interesting fact is that the error for Runge-Kutta methods depends on the form of the equation as well as on the solution itself.