MATH 335: Numerical Analysis

Problem Set 16, Final version

Due Date: Mon., Apr. 20, 2009

Read Chapter 10 from Kharab & Guenther, along with your in-class notes.

10.1.6 Suppose we use the expression

$$h f(a+h) + \frac{1}{2} h^2 f'(a) \qquad \text{to approximate} \qquad \int_a^{a+h} f(x)\,dx\ .$$

Find an expression for the (truncation) error.

10.1.20 Estimate the error involved in approximating

$$\int_1^3 \frac{1}{\sqrt{1+x^3}}\,dx$$

using the composite trapezoid rule with $n = 100$.

★33 Consider the definite integral $\int_0^1 \sqrt{x}\,dx$.

(a) (Ex. 10.1.10) Approximate this integral using the composite trapezoid rule with $h = 1/2, 1/4, 1/8, \ldots$. Do you get the expected rate of convergence?

(b) (Ex. 10.2.13) Approximate this integral using the composite Simpson 1/3 rule with $h = 1/2, 1/4, 1/8, \ldots$. Do you get the expected rate of convergence?

10.2.20 Determine the number of subintervals n required to approximate

$$\int_0^2 e^{2x} \sin(3x)\,dx$$

with an error less than 10^{-4} using

(a) the composite trapezoidal rule.

(b) the composite Simpson 1/3 rule.

10.3.8 Use Romberg's method to compute the integral $\int_0^4 f(x)\,dx$ where $f(x)$ is defined by the following table. Do we need all the values to be used?

x	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	-4271	-2522	-499	1795	4358	7187	10279	13633	17246

★34 (a) Download the file `http://web.cecs.pdx.edu/~gerry/nmm/mfiles/integrate/GLNodeWt.m` and place it in your Octave working directory. This routine may be used to generate weights and nodes for any n like those found in Table 10.5, p. 356. (I know little else about this routine, except that it seems to accompany the text **Numerical Methods with Matlab: Implementations and Applications**, by Gerald Recktenwald, available from Prentice-Hall.) You should get comfortable with its use. Then write a routine with declaration

```
function intVal = glQuad(f, a, b, n)
```

which uses Gauss-Legendre quadrature on n nodes to approximate the value of $\int_a^b f(x)\,dx$. (I expect you will want to employ the `GLNodeWt()` routine during the process.)

(b) (Ex. 10.4.5) Test your routine by computing

$$\int_0^{0.999} \tan\left(\frac{\pi}{2}x\right) dx$$

using Gauss-Legendre quadrature with $n = 4$ nodes. (The answer is in the back.)