# MATH 335: Numerical Analysis

## Problem Set 14, Final version

## Due Date: Mon., Apr. 6, 2009

Read the handout titled "Exponential Polynomial Interpolation and the Fast Fourier Transform".

$\star$26 We have defined an $n$-inner product between vectors $\mathbf{u} = (u_0, \ldots, u_{n-1})$, $\mathbf{v} = (v_0, \ldots, v_{n-1})$ of $\mathbb{C}^n$ by

$$\langle \mathbf{u}, \mathbf{v} \rangle = \frac{1}{n} \sum_{j=0}^{n-1} u_j \overline{v_j} .$$

In like manner, we could define something like an inner product between continuous functions (different from the inner product we already have). Specifically, let $x_0$, $x_1$, $\ldots$, $x_{n-1}$ be points in the interval $[a, b]$. When both $f, g\colon [a, b] \to \mathbb{C}$ are continuous (the continuity is sufficient, though not necessary), we may define

$$\langle\langle f, g \rangle\rangle_n := \frac{1}{n} \sum_{j=0}^{n-1} f(x_j) \overline{g(x_j)} .$$

Note that $\langle\langle \cdot, \cdot \rangle\rangle_n$ has these properties that an inner product ought to have:

- $\langle\langle f_1 + f_2, g \rangle\rangle_n = \langle\langle f_1, g \rangle\rangle_n + \langle\langle f_2, g \rangle\rangle_n$, for each $f_1$, $f_2$, $g \in C([a, b])$,
- $\langle\langle f, g \rangle\rangle_n = \overline{\langle\langle g, f \rangle\rangle_n}$, for each $f$, $g \in C([a, b])$,
- $\langle\langle \alpha f, g \rangle\rangle_n = \alpha \langle\langle f, g \rangle\rangle_n$, for each $f$, $g \in C([a, b])$, $\alpha \in \mathbb{C}$.

(a) Equipped with this definition, we might go on to define

$$|||f|||_n := \sqrt{\langle\langle f, f \rangle\rangle_n} .$$

Determine whether $||| \cdot |||_n$ is a *norm*—that is, satisfies all the requirements of a norm—on $C([a, b])$.

(b) Suppose we are dealing with a function $f\colon [0, 2\pi] \to \mathbb{C}$ that is continuous. Moreover, assume that, for some positive integer $n$, $f \in \operatorname{span}(\{E_0, \ldots, E_{n-1}\})$, where $E_k(x) = e^{ikx}$, which means there are coefficients $c_k$, $k = 0, \ldots, n-1$ such that

$$f(x) = \sum_{k=0}^{n-1} c_k E_k(x) , \qquad \text{for all } x \in [0, 2\pi] .$$

Show that, if we take evenly spaced $x_j = 2\pi j/n$, for $j = 0, \ldots, n-1$, then $c_j = \langle\langle f, E_j \rangle\rangle_n$, for $j = 0, \ldots, n-1$.

★27 The climatological phenomenon *el Niño* results from changes in atmospheric pressure in the southern Pacific ocean. The "Southern Oscillation Index" is the difference in atmospheric pressure between Easter Island and Darwin, Australia, measured at sea level at the same moment. The text file `elnino.dat` (click on the red text to see the file) contains values of this index measured on a monthly basis over the 14-year period 1962 through 1975.

Your assignment is to carry out an analysis similar to the *sunspot* example (done in class) on this *el Niño* data. The unit of time is one month instead of one year. You should find there is a prominent cycle with a period of 12 months, and second, less prominent, cycle with a longer period. This second cycle shows up in about three of the Fourier coefficients, so it is hard to measure its length, but see if you can make an estimate.