MATH 335: Numerical Analysis

Problem Set 5, Final version

Due Date: Thurs., Feb. 19, 2009

Read Sections 3.2, 3.3 and 3.5 in Kharab & Guenther.

3.3.5 For each of the following functions, locate an interval on which fixed-point iteration will converge.

(a) $x = 1 + 0.2 \sin x$

(b) $x = 1 - (1/4)x^2$.

3.3.12 Consider the fixed-point problem

$$x = \frac{1}{2}\tan\left(\frac{\pi x}{2}\right)$$

on the open interval $0 < x < 1$. Explain both graphically and analytically why fixed-point iteration $x_{n+1} = (1/2)\sin(\pi x_n/2)$, $n = 0, 1, 2, \ldots$ does not converge to the unique fixed point $\alpha = 1/2$ in the interval $(0, 1)$ for any starting point $x_0 \in (0, 1)$, $x_0 \neq 1/2$.

3.3.14 Consider the fixed-point iteration $p_{n+1} = g(p_n)$ when the function $g(x) = 2(x-1)^{1/2}$ for $x \geq 0$ is used. Plot the function and the line $y = x$ on the same plot and determine how many fixed points exist. Iterate with $p_0 = 1.5$ and with $p_0 = 2.5$. Explain the result in the light of the fixed-point theorem.

3.2.7 Find the real roots of $f(x) = x^3 - 2.56x^2 - 34.6x + 112.5$ using the method of false position.

3.5.5 Use Newton's method to compute a zero of the function

$$f(x) = x^5 - 3x^3 - 5x + 4 \; .$$

3.5.17 In this problem we will find the roots of $\lambda x = \tan x$. This equation describes the states of a quantum-mechanical particle in a rectangular box with finite walls. The equation cannot be solved analytically and one needs to use numerical root-finding. You are asked to analyze this equation for three positive values of λ: one $\lambda < 1$, another $\lambda = 1$, and third $\lambda > 1$. (You choose the values of λ yourself.)

(a) (Analytical). What happens to the roots as $\lambda \to 1$?

(b) Plot the functions λx and $\tan x$ on the same graph and find out approximately where the roots are.

(c) Find an approximate formula for x_{n+1} using Newton's method.

(d) Use the (supplied) function `newton.m` to compute a root of the equation in $(\pi/2, 3\pi/2)$.

★4 Let

$$g(x) = \frac{x + \frac{A}{x}}{2} .$$

This g has been employed, along with fixed-point iteration, to compute $\sqrt{A}$.

(a) Use Newton's method to derive the formula

$$g(x) = \frac{(n-1)x - \frac{A}{x^{n-1}}}{n}$$

(of which the above is a special case) of a function that could be used in a fixed-point iteration to find $\sqrt[n]{A}$, $n = 2, 3, \ldots$, $A > 0$. (What is special about this formula is that it employs only the simple operations $+$, $-$, $\times$, $\div$ to calculate roots.)

(b) Apply this method to find the 4th root of 7.