

MATH 335: Problem Set #3

Read Sections 2.2–2.3, 9.1 in Kharab & Guenther.

- ★1 (a) Solve the quadratic equation

$$x^2 - 12.4x + 0.494 = 0,$$

by evaluating the quadratic formula using three-digit decimal arithmetic and rounding. The exact roots rounded to 6 digits are 0.0399675 and 12.3600.

- (b) You should observe *loss of significance* in one of your computed zeros of part (a). If we consider the 6-digit answers provided as the “true” ones, what is this solution’s relative error? Following Definition 2.2 in Kharab & Guenther (p. 33), to how many significant digits does your computed root approximate the true one? Explain what aspect of the coefficients in the quadratic equation to be solved led to this loss of significance.
- (c) Show that the roots r_1, r_2 of a quadratic equation $ax^2 + bx + c = 0$ satisfy $r_1 r_2 = c/a$.
- (d) While there is loss of significance in one solution of the quadratic equation in (a), it does not occur in the other. Write an OCTAVE/MATLAB function which accepts as a vector $[a \ b \ c]$ the coefficients of a quadratic function $f(x) = ax^2 + bx + c$, computes the root that has larger absolute value, and then obtains the other root using the formula in part (c). You may assume that the roots are distinct real numbers—i.e., that $b^2 - 4ac > 0$. (Hand in a printout of your function.)
- (e) As in part (a), use 3-digit decimal arithmetic to solve the quadratic equation

$$x^2 - 12.4x + 38.4 = 0,$$

whose exact roots rounded to 6 digits are 6.00000 and 6.40000. You should once again observe loss of significance. Does our method in part (d) for dealing with loss of significance help here? If not, what aspect of this situation makes it different?

2.2.5 If x_1, x_2 are approximations to X_1, X_2 with errors $\varepsilon_1, \varepsilon_2$, show that the relative error

$$\frac{X_1 X_2 - x_1 x_2}{X_1 X_2}$$

of the product $X_1 X_2$ is approximately the sum $\frac{\varepsilon_1}{X_1} + \frac{\varepsilon_2}{X_2}$.

★2 In class we showed that, under suitable assumptions on f and h , we have

$$f'(x) = \frac{1}{2h}[f(x+h) - f(x-h)] - \frac{h^2}{12}[f'''(\xi_1) + f'''(\xi_2)],$$

where ξ_1 lies between x and $x+h$, and ξ_2 lies between x and $x-h$.

- (a) What, exactly, are these assumptions on f and h ?
- (b) Supposing that f''' is suitably continuous, show that the truncation error term $(-h^2/12)[f'''(\xi_1) + f'''(\xi_2)]$ may be replaced by a single term $(-h^2/6)f'''(\xi)$. (Hint: Use the IVT.) In what interval could we assume this new number ξ resides?

9.1.3 Derive the following approximate formulas:

- (a) $f'(x) \approx \frac{1}{4h}[f(x+2h) - f(x-2h)]$
- (b) $f'(x) \approx \frac{1}{2h}[4f(x+h) - 3f(x) - f(x+2h)]$

9.1.3 Use Taylor's series to derive the following approximation formula for the first derivative of f :

$$f'(x) \approx \frac{1}{6h}[2f(x+3h) - 9f(x+2h) + 18f(x+h) - 11f(x)].$$

9.1.9 Show that the approximation formula in Exercise 9.1.8 has an error of $O(h^3)$.