MATH 333: Partial Differential Equations

Problem Set 1, Final version

Due Date: Mon., Sept. 14, 2009

Read Chapter 1 from the Olver text.

{★1} (The brackets {} indicate this problem is optional.) Download and install a copy of OCTAVE on your personal computer. (Windows users who do not already have `cygwin` installed on your computer—and you know it if you do—should choose the *Octave-Forge Files* link.) To run OCTAVE (after installing it) in a unix/linux/macintosh system, you can type `octave` in a terminal window. I believe in a Microsoft Windows system you will have an icon somewhere off the "Start" menu that says "Octave". Once you have it running, take a look at the tutorial found at `http://www-mdp.eng.cam.ac.uk/web/CD/engapps/octave/octavetut.pdf`. Go through the first 5 sections (up to "OCTAVE programming I: Script files") of the tutorial, executing the commands in your own OCTAVE window as you go, in order to get an initial sense of the software interface OCTAVE provides. I suspect that we may begin to have some discrepancies between what the tutorial says works and what actually does in Section 5 ("Plotting graphs"), as the tutorial is now a few years old. If you are having difficulties, determine what they are as specifically as possible, bring them up in class, and we will see how common such difficulties are and/or whether someone has figured out a workaround.

1.1 Classify each of the following differential equations as (i) ordinary or partial; and, (ii) equilibrium or dynamic. (iii) Then write down its order:

(a) $\dfrac{du}{dx} + xu = 1,$

(b) $\dfrac{\partial u}{\partial t} + u\,\dfrac{\partial u}{\partial x} = x,$

(c) $u_{tt} = 9u_{xx},$

(d) $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial u}{\partial x},$

(e) $\dfrac{d^2 u}{dt^2} + 3u = \sin t,$

(f) $-\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial y^2} = x^2 + y^2,$

(g) $u_{xx} + u_{yy} + u_{zz} + (x^2 + y^2 + z^2)u = 0,$

(h) $u_{xx} = x + u^2,$

(i) $\dfrac{\partial u}{\partial t} + \dfrac{\partial^3 u}{\partial x^3} + u\dfrac{\partial u}{\partial x} = 0,$

(j) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y\,\partial z} = u,$

(k) $u_{tt} = u_{xxxx} + 2u_{xxyy} + u_{yyyy}.$

$\star$2 For what values of a and b will $u(t,x) = e^{at}\sin(bx)$ solve the heat equation $u_t = \gamma u_{xx}$, where γ is a constant?

$\star$3 Find a function $u = u(t,x)$ that satisfies the PDE

$$u_{xx} = 0, \qquad 0 < x < 1, \quad t > 0,$$

subject to the boundary conditions

$$u(t,0) = t^2, \qquad u(t,1) = 1, \qquad t > 0.$$

$\star$4 Find the general solution to the equation

$$u_{xt} + 3u_x = 1.$$

Hint: Let $v = u_x$ and solve the resulting equation for v; then find u.

$\star$5 Show that the nonlinear equation

$$u_t = u_x^2 + u_{xx}$$

can be reduced to the one-dimensional heat equation by changing the dependent variable via $w = e^u$.

$\star$6 In a three-dimensional spherical region D free of charges, the static electric field vector $\mathbf{E}$ satisfies the two Maxwell equations

$$\operatorname{div}\mathbf{E} = \nabla\cdot\mathbf{E} = 0, \qquad \operatorname{curl}\mathbf{E} = \nabla\times\mathbf{E} = \mathbf{0}.$$

(The electric field is produced, say, by electrical charges outside of D.) Let $V = V(x,y,z)$ be the electrical potential (so $\nabla V = \mathbf{E}$; multivariable calculus may be used to show that such a potential V exists). Show that the static electric field potential V satisfies Laplace's equation; that is, $\Delta V = 0$.