

MATH 333: Partial Differential Equations

Project 1, Due Date: Fri., Oct. 20, 2006

In this project you will study in greater depth some of the phenomena which can arise in first-order PDEs. In particular, you will study quasilinear PDEs (i.e., the coefficients of highest-order derivatives of the unknown (density) function ρ depend only on x , t , and lower-order derivatives of ρ) of the form

$$\begin{aligned}\frac{\partial \rho}{\partial t} + c(\rho) \frac{\partial \rho}{\partial x} &= 0, & x \in \mathbb{R}, t > 0, \\ \rho(x, 0) &= f(x), & x \in \mathbb{R}.\end{aligned}\tag{1}$$

You are to consult at least 3 sources besides the internet (likely PDE texts), and using these you are to write out a set of stand-alone notes which could be used some years from now by you to get up to speed on the concepts of how one is to understand solutions of problem (1) in various cases. You should explain what a compression wave is and how such a thing might arise physically. You should explain the relationship between a compression wave and a shock wave—in particular, why, in certain settings, a shock wave seems to be more physically realisable, what criterion one might use for determining the velocity of a shock, and the solution that arises from that criterion. You should also discuss rarefaction waves, and their relationship to the aforementioned ones. I would also like to see these notes take up a particular application that might exhibit these kinds of solutions. One possible instance might be traffic flow. The notes might contain an explanation of why it would make sense for a model of traffic flow to take the form (1), and whether there are simple traffic scenarios in which the types of waves (rarefaction, shock, compression) already explained might arise. A particularly helpful resource in this vein is **Applied Partial Differential Equations with Fourier Series and Boundary Value Problems, 4th Ed.**, by Richard Haberman. While I am not an expert in this area, you are welcome to consult with me as you work through this.