

MATH 333: Partial Differential Equations  
Project 5, Due Date: Fri., Oct. 13, 2006

Consider the ordinary differential equation

$$\frac{d^2y}{dx^2} - xy = 0, \quad -\infty < x < \infty.$$

Use Fourier transforms to show solutions of this ODE are scalar multiples of the Airy function

$$\text{Ai}(x) := \frac{1}{2\pi} \int_{-\infty}^{\infty} \cos(\omega^3/3 + \omega x) d\omega.$$

(You may assume that integrals over  $(-a, a)$ , even when  $a = \infty$ , are zero when the integrand is an odd function.) Next, solve the Cauchy problem for the linearized Korteweg-deVries (KdV) equation

$$\begin{aligned} u_t + ku_{xxx} &= 0, & x \in \mathbb{R}, & t > 0, \\ u(x, 0) &= f(x), & x \in \mathbb{R}, & \end{aligned}$$

where  $k > 0$  is constant, expressing your answer in terms of the Airy function. (The KdV equation, in its nonlinear form, is a model for waves on shallow water surfaces.) Finally, assuming a Gaussian initial condition  $f(x) = e^{-x^2}$ , sketch a solution profile at time  $t = 1$ .