Consider the ordinary differential equation
\[
\frac{d^2y}{dx^2} - xy = 0, \quad -\infty < x < \infty.
\]

Use Fourier transforms to show solutions of this ODE are scalar multiples of the Airy function
\[
Ai(x) := \frac{1}{2\pi} \int_{-\infty}^{\infty} \cos(\omega^3/3 + \omega x) \, d\omega.
\]

(You may assume that integrals over \((-a, a)\), even when \(a = \infty\), are zero when the integrand is an odd function.) Next, solve the Cauchy problem for the linearized Korteweg-deVries (KdV) equation
\[
\begin{align*}
  u_t + ku_{xxx} &= 0, \quad x \in \mathbb{R}, \ t > 0, \\
  u(x, 0) &= f(x), \quad x \in \mathbb{R},
\end{align*}
\]
where \(k > 0\) is constant, expressing your answer in terms of the Airy function. (The KdV equation, in its nonlinear form, is a model for waves on shallow water surfaces.) Finally, assuming a Gaussian initial condition \(f(x) = e^{-x^2}\), sketch a solution profile at time \(t = 1\).