

MATH 333: Partial Differential Equations

Problem Set 7, Final version

Due Date: Mon., Oct. 23, 2006

1. Do Exercise 2.29. If you look over the help for *Octave's* `meshgrid` command, you might be able to produce your matrix T while saving a few lines of code. (Don't worry about it if you're comfortable with a wordier approach.)
2. Let L be a linear operator on a vector space V . In particular, assume $Lv \in V$ for each $v \in V$ (that is, the output from L lies in the same space as the input to it, which is nearly always what mathematicians mean when they use the phrase "operator on V "). Suppose that w is an eigenvector of L (i.e., that w is not the zero vector in V , and that there exists a scalar λ for which $Lw = \lambda w$). Prove that, for any scalar c and positive integer m , w is also an eigenvector of the operator $(I + cL)^m$, where I denotes the identity operator on V —that is, $Iv = v$ for all $v \in V$. This result gets used (without being proved) in Chapter 4.
3. Suppose that A is a symmetric operator on an inner product space V equipped with inner product $\langle \cdot, \cdot \rangle$; that is, given any two vectors u, v in V , $\langle Au, v \rangle = \langle u, Av \rangle$. Being symmetric, the eigenvalues of A are real. (You may assume this as fact for this problem. A proof, specific to the case where $A = -d^2/dx^2$, is sketched out in Exercise 2.28.) Prove that, if u, v are eigenvectors of A corresponding to *distinct* eigenvalues λ, μ respectively, then u and v are orthogonal. (Note: A proof that applies to this general statement has been given in roughly two places in Chapter 2.)

As we have seen, a vector space may allow for more than one definition of an inner product. Does the result you just proved seem to be tied to the particular inner product being used? Explain.