1. Consider the derivative operator \( A = d^4/dx^4 \) acting on some subset of functions \( \phi \) in \( L^2(0, 1) \) which satisfy \( \phi(0) = \phi'(0) = 0 = \phi(1) = \phi'(1) \). This operator arises in the Bernoulli-Euler model of transverse vibrations of beam clamped at both ends. Show that \( A \) is symmetric, positive definite.

2. Read through Exercise 2.16. Do parts (b) and (c).

3. Do Exercise 2.17.

4. Do Exercise 2.18.

5. Let \( \alpha \in \mathbb{R} \) be fixed, and consider the linear differential operator given by

\[ Lu := -u'' + \alpha u, \]

restricted to the domain \( C_0^2(0, 1) \). By definition, for a certain (possibly nonreal) number \( \lambda \) to be an eigenvalue of a linear operator \( L \), there needs to exist a function \( u \in C_0^2(0, 1) \) not identically zero and satisfying the equation \( Lu = \lambda u \).

(a) Without actually finding any eigenvalues, show that an eigenvalue \( \lambda \) of this particular operator must be real and satisfy \( \lambda > \alpha \).

(b) Now do Exercise 2.24.

6. This exercise is a follow-up to problem 10(a) on Problem Set 1. Suppose \( V \) is an \( n \)-dimensional vector space (it need not, however, be \( \mathbb{R}^n \)) with basis \( \{u_1, u_2, \ldots, u_n\} \). This means that the \( u_j \) are linearly independent

\[ c_1u_1 + \cdots + c_nu_n = 0 \quad \text{implies} \quad c_1 = \cdots = c_n = 0, \]

and \( \text{span} \ V \) i.e., every \( v \in V \) can be written as a linear combination of the \( u_j \) or, to say it yet another way, for all \( v \in V \), there exist scalars \( c_1, \ldots, c_n \) (specific to the choice of \( v \)) such that

\[ v = c_1u_1 + c_2u_2 + \cdots + c_nu_n. \]

(a) Show that no \( v \in V \) can be written as a linear combination of the \( u_j \) in two different ways. That is, show if

\[ v = \sum_{j=1}^{n} c_ju_j \quad \text{and} \quad v = \sum_{j=1}^{n} d_ju_j, \]

then \( c_1 = d_1, \ c_2 = d_2, \ldots, \ c_n = d_n \).
(b) Assume now that $V$ is an inner product space with inner product $\langle \cdot, \cdot \rangle$, and that the $u_j$ are mutually orthogonal. Under these conditions, show that the scalars $c_j$ are given by

$$ c_j = \frac{\langle v, u_j \rangle}{\langle u_j, u_j \rangle}. $$