

MATH 333: Partial Differential Equations

Problem Set 4, Final version

Due Date: Wed., Oct. 4, 2006

1. Do Exercise 12.2(a).

2. Do Exercise 12.7.

3. Solve the boundary value problem

$$\begin{aligned}u_{xx} + u_{yy} &= 0, & x \in \mathbb{R}, & y > 0 \\u(x, 0) &= 1, & |x| \leq \ell; & \quad u(x, 0) = 0, & |x| > \ell,\end{aligned}$$

where $\ell > 0$ is a constant.

4. Do Exercise 2.1. Here the phrase “unit interval” refers to the interval $(0, 1)$.

5. (a) Let $n > 0$ be a fixed integer. Use Taylor’s theorem (explaining why it is applicable) to derive the following expression for $\sin x$:

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots + \frac{\sin^{(n)}(0)}{n!}x^n + \frac{\sin^{(n+1)}(c)}{(n+1)!}x^{n+1},$$

where $\sin^{(n)}(x)$ denotes the n th derivative $d^n/dx^n(\sin x)$, and c is a number between x and 0.

(b) The final term in the above expression, the one involving the power x^{n+1} , is called the *remainder term*. Suppose you wish to approximate $\sin x$ by the polynomial in x that results by omitting the remainder term—that is,

$$\sin x \approx x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots + \frac{\sin^{(n)}(0)}{n!}x^n.$$

If you know you are going to stick to x -values in the interval $[-5, 5]$, what is the smallest n can be to guarantee the error in the above approximation does not exceed 10^{-5} ?

6. Consider how you would convert the solution (2.7) of problem (2.1) into the form (2.8)–(2.9); that is, consider how you would find the Green’s function. Then do Exercise 2.5.

7. Do Exercise 2.6.

8. Read through Exercise 2.8. If you had to summarize what you know of Green’s functions to this point, what would you say about them? What parallels do you see between the role of Green’s functions in the problems you have worked from Chapter 2 and the role of the *fundamental solution* $S(x, t)$ of the heat equation given in (12.33), p. 377 (there with $k = 1$) in solving heat problems of the form (12.20)?