

MATH 143

Inference practice problems

1. An automobile manufacturer wants to know what percentage of its customers are dissatisfied with the performance of their newly-purchased vehicle.
 - (a) Name the procedure we have learned best suited for determining the answer to this question.
 - (b) Without any real estimate of the answer to this question, how many customers should be polled so that we are 95% sure our answer is within 3% of the correct one?
 - (c) A random sample of 252 customers yields 53 who are dissatisfied. Construct a 95% CI for the appropriate population parameter. What population parameter is this? Just what are we 95% sure about?
 - (d) Given the number of things that affect customer satisfaction and are *out* of the manufacturer's control, it has been determined that 85% customer satisfaction is an acceptable figure. Does the sample data of part (c) provide convincing evidence that customer satisfaction is at unacceptably low levels?
2. The following data give for each day of a one-week period the number of crimes reported to a certain police station. In general, is crime distributed uniformly across days of the week?

Day	Su	Mo	Tu	We	Th	Fr	Sa	Total
# of crimes	48	38	31	40	39	40	44	280

3. After studying a sample of 500 students at State University, a research investigating the study habits of college students concludes that college students study an average of 10.25 hours, give or take 1.5 hours, per week during the academic year.
 - (a) Which type of statistical inferential procedure was likely used to determine this figure?
 - (b) Can you see any potential problems in the conclusion the researcher drew?
4. To see if there is a significant difference in grades given in English 100 and Religion 103, 20 students who had taken both courses the first semester of their freshman year were randomly selected and their grades were recorded. Their grades are
 - (a) Name the procedure we have learned best suited for determining the answer to this question.
 - (b) State the null and alternative hypotheses.
 - (c) For each student in the sample, the difference in grade is taken by subtracting the religion grade from the english one. The average of these differences is 0.17 with standard deviation 0.32. Determine the P -value associated with this sample mean under the null hypothesis. Is the evidence significant at the 1% level to reject H_0 ?

5. A random sample of 54 aircraft air-conditioning units in for repair is selected and monitored to see the number of hours of use before another repair is needed. Here is a histogram of the data.

- (a) If the question one wishes to answer is whether there is strong evidence that these AC units have a mean operating time of more than 50 hours between repairs, then what kind of procedure is appropriate? What are the hypotheses?
- (b) Is it accurate to say that the histogram above displays the population distribution?
- (c) Should we have any reservations about using the procedure you mentioned in part (a) for this data?

6. A chemical engineer is designing the production process for a new product. The chemical reaction that produces the product may have higher or lower yield, depending on the temperature and the stirring rate in the vessel in which the reaction takes place. The engineer decides to investigate the effects of combinations of two temperatures (50°C and 60°C) and three stirring rates (60 rpm, 90 rpm and 120 rpm) on the yield of the process. Two batches of the product will be processed at each combination of temperature and stirring rate.

- (a) What are the variables involved? Designate each as explanatory or response.
- (b) How many factors are there in this experiment?
- (c) What type of statistical procedure is likely applied to analyze the experimental results?
- (d) How many total degrees of freedom are there?

7. Abel and Ben both own Christmas tree farms. Abel claims that his trees are taller, on average, than Ben's. A random sample of trees were selected from each farm, and their heights were recorded.

- (a) May we consider the two samples to be independent?

- (b) What type of statistical procedures are available to us for answering questions about the mean difference in heights of trees from the two farms?
- (c) Suppose a 99% confidence interval for the difference between the mean height of trees on Abel and Ben's farms ($\mu_A - \mu_B$) is given by (0.2, 0.7). Are Abel's trees significantly taller than Ben's? If so, significant at which of our usual significance levels?
- (d) What was the difference in sample means ($\bar{x}_A - \bar{x}_B$), given the information in part (c)?

8. In constructing a confidence interval,

- (a) If I increase the level of confidence, I will _____ the width of the interval.
- (b) If I increase the sample size n , I will _____ the width of the interval.
- (c) If (somehow) I increase the standard deviation, I will _____ the width of the interval.

9. A researcher wants to see if boys and girls begin walking at the same age. She takes random samples of each. Of 81 girls, the mean age at which they begin walking is 11.9 months with standard deviation 3.4 months. For the 88 boys in her sample, the numbers are 12.2 and 4 respectively.

- (a) State null and alternative hypotheses for the question she is trying to answer.
- (b) Determine the P -value associated with this data under the assumption that the null hypothesis is true. Explain what this P -value indicates.
- (c) Find a 95% CI for the difference in mean age ($\mu_B - \mu_G$) at which children from the two populations begin walking. Based on the result of part (b), what two numbers can you be sure are in this confidence interval before you even calculate it?

10. A study was done to see the role piano lessons play in spatial-temporal reasoning. A pre and post-test to assess spatial-temporal reasoning was given to children who, in the time between these tests, were given piano lessons. Three other groups were given the same tests: 10 children who received singing lessons, 20 who received computer instruction, and the 14 who received no special lessons of any kind.

- (a) What kind of (single) statistical test may be used to assess whether the mean difference between post and pre-test scores is the same for all four groups of children?
- (b) Which F -distribution (i.e., which df -numerator, df -denominator) would you consult in the process of determining a P -value for the test statistic associated with your answer to part (a)?
- (c) Suppose the test statistic $F = 9.24$. What can you say about the associated probability of getting an F -value at least this extreme under the null hypothesis?
- (d) If the null hypothesis has been rejected, there is still the need to run individual comparisons between group pairs to see which means are actually different. What kind of test is each one of these individual comparisons? If we want a family significance level to be 5%, why is it the individual comparisons must be run at adjusted significance levels?

- (e) Here is the *Stata* output for a Bonferroni multiple comparison between groups. Which groups are significantly different at the 5% level? Which group has the largest improvement between pre and post-test scores? Rank the groups according to their sample means, from the group whose change in spatio-temporal reasoning was the smallest up to the group whose change was largest.

Row Mean-				
Col Mean	Computer	None	Piano	
None	.335714			
	1.000			
Piano	3.16765	2.83193		
	0.001	0.010		
Singing	-.75	-1.08571	-3.91765	
	1.000	1.000	0.001	

11. In a study to see if a prenotification letter had an effect on response rate to a survey, 5018 physicians were sent a prenotification letter prior to being sent a survey. Of those, 2570 did respond to the survey. At the same time, 5029 physicians were mailed the survey with no prenotification letter, and 2645 responded.
- (a) What null and alternative hypotheses are of interest here? What two statistical procedure are appropriate for testing this null hypothesis?
- (b) Carry out both tests.