Linear Algebra Formulas

Angle θ between vectors $\mathbf{u}, \mathbf{v}$ satisfies: $\cos\theta = \frac{\langle \mathbf{u}, \mathbf{v} \rangle}{|\mathbf{u}||\mathbf{v}|}$

Normal equations for $\mathbf{A}\mathbf{x} = \mathbf{b}$: $(\mathbf{A}^T\mathbf{A})\mathbf{x} = \mathbf{A}^T\mathbf{b}$

Statistics Formulas

Means and Variances

	Mean	Variance
Sample (of size n)	$\overline{x} = \frac{1}{n}\sum_i x_i$	$s^2 = \frac{1}{n-1}\sum_i (x_i - \overline{x})^2$
Random variable X		
discrete X		
General	$E(X) = \sum_x x f_X(x)$	$\mathrm{Var}(X) = \sum_x (x-\mu_X)^2 f_X(x) = E([X - E(X)]^2) = E(X^2) - [E(X)]^2$
$X \sim \mathsf{Binom}(n, \pi)$	$E(X) = n\pi$	$\mathrm{Var}(X) = n\pi(1-\pi)$
$X \sim \mathsf{Hyper}(m, n, k)$	$E(X) = \frac{km}{m+n}$	$\mathrm{Var}(X) = k\left(\frac{m}{m+n}\right)\left(\frac{n}{m+n}\right)\left(\frac{m+n-k}{m+n-1}\right)$
continuous X		
General	$E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx$	$\mathrm{Var}(X) = \int_{-\infty}^{\infty} [x - E(X)]^2 f_X(x)\,dx = E([X - E(X)]^2) = E(X^2) - [E(X)]^2$
$X \sim \mathsf{Unif}(a, b)$	$E(X) = \frac{1}{2}(a+b)$	$\mathrm{Var}(X) = \frac{1}{12}(b-a)^2$
$X \sim \mathsf{Exp}(\lambda)$	$E(X) = 1/\lambda$	$\mathrm{Var}(X) = 1/\lambda^2$
$X \sim \mathsf{Norm}(\mu, \sigma)$	$E(X) = \mu$	$\mathrm{Var}(X) = \sigma^2$

$100(1-\alpha)$% Confidence Intervals

For $T \sim t(\nu)$ choose $t_{\beta,\nu}$ so that $P(T > t_{\beta,\nu}) = \beta$. The 2-sided $100(1-\alpha)$% CI is

$$(\text{estimate}) \pm t^*(\text{std. error}),$$

with $t^* = t_{\alpha/2,\nu}$, and the degrees of freedom ν chosen appropriately (see below).

What estimating?	Sample size	Estimate	std. error	ν
μ_X	n	$\overline{x}$	$\frac{s}{\sqrt{n}}$	$n-1$
$\mu_X - \mu_Y$	m, n (resp.)	$\overline{x} - \overline{y}$	$\left(\frac{s_X}{m} + \frac{s_Y}{n}\right)^{1/2}$	$\dfrac{\left(\frac{s_X^2}{m} + \frac{s_Y^2}{n}\right)^2}{\frac{(s_X^2/m)^2}{m-1} + \frac{(s_Y^2/n)^2}{n-1}}$
β_1	n	b_1	$s_{b_1} = \sqrt{\frac{\mathrm{MSE}}{\sum_i (x_i - \overline{x})^2}}$	$n-2$

Vector Calculus Formulas

Fundamental theorems (main result)

FT of Line Integrals:	If $\mathbf{F} = \nabla f$, and the curve C has endpoints A and B, then $\int_C \mathbf{F} \cdot d\mathbf{r} = f(B) - f(A)$.
Green's Theorem:	$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA \;=\; \oint_C \mathbf{F} \cdot d\mathbf{r}$ (circulation-curl form)
Stokes' Theorem:	$\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\, d\sigma \;=\; \oint_C \mathbf{F} \cdot d\mathbf{r}$, where C is the edge curve of S
Green's Theorem:	$\iint_R \nabla \cdot \mathbf{F}\, dA \;=\; \oint_C \mathbf{F} \cdot \mathbf{n}\, ds$ (flux-divergence form)
Divergence Theorem:	$\iiint_R \nabla \cdot \mathbf{F}\, dV \;=\; \oiint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma$

Differential elements

Along a parametrized curve $\mathbf{r}(t)$, $t \in [a, b]$, we have $ds = \left\| \frac{d\mathbf{r}}{dt} \right\| dt$.
Along a parametrized surface $\mathbf{r}(u, v)$, $(u, v) \in D$, we have $d\sigma = \left\| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right\| du\, dv$.

Curl and divergence

For a continuously differentiable 3D vector field $\mathbf{F}(x, y, z) = M(x, y, z)\mathbf{i} + N(x, y, z)\mathbf{j} + P(x, y, z)\mathbf{k}$,

$$\text{curl}\,\mathbf{F} \;=\; \nabla \times \mathbf{F} \;:=\; \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} \right)\mathbf{i} + \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x} \right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)\mathbf{k}\,.$$

$$\text{div}\,\mathbf{F} \;=\; \nabla \cdot \mathbf{F} \;:=\; \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot (M, N, P) \;=\; \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}\,.$$

Other 3D spatial coordinates

$$\begin{aligned}
r &= \sqrt{x^2 + y^2} = \rho \sin\phi, \\
\rho &= \sqrt{x^2 + y^2 + z^2}, \\
x &= r\cos\theta = \rho \sin\phi \cos\theta, \\
y &= r\sin\theta = \rho \sin\phi \sin\theta, \\
z &= \rho\cos\phi\,, \\
dV &= dz\,dy\,dx \\
&= r\,dz\,dr\,d\theta \\
&= \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta\,.
\end{aligned}$$