

Vector Calculus Formulas

Fundamental theorems (main result)

FT of Line Integrals:	If $\mathbf{F} = \nabla f$, and the curve C has endpoints A and B , then $\int_C \mathbf{F} \cdot d\mathbf{r} = f(B) - f(A).$
Green's Theorem:	$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA = \oint_C \mathbf{F} \cdot d\mathbf{r}$ (circulation-curl form)
Stokes' Theorem:	$\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} \, d\sigma = \oint_C \mathbf{F} \cdot d\mathbf{r}$, where C is the edge curve of S
Green's Theorem:	$\iint_R \nabla \cdot \mathbf{F} \, dA = \oint_C \mathbf{F} \cdot \mathbf{n} \, ds$ (flux-divergence form)
Divergence Theorem:	$\iiint_R \nabla \cdot \mathbf{F} \, dV = \iiint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma$

Differential elements

Along a parametrized curve $\mathbf{r}(t)$, $t \in [a, b]$, $ds = \left\| \frac{d\mathbf{r}}{dt} \right\| dt$.

Along a parametrized surface $\mathbf{r}(u, v)$, $(u, v) \in D$, $d\sigma = \left\| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right\| du \, dv$.

Curl and divergence

For a continuously differentiable 3D vector field $\mathbf{F}(x, y, z) = M(x, y, z)\mathbf{i} + N(x, y, z)\mathbf{j} + P(x, y, z)\mathbf{k}$,

$$\text{curl } \mathbf{F} = \nabla \times \mathbf{F} := \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} \right) \mathbf{i} + \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x} \right) \mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \mathbf{k}.$$

$$\text{div } \mathbf{F} = \nabla \cdot \mathbf{F} := \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot (M, N, P) = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}.$$

Other 3D spatial coordinates

$$r = \sqrt{x^2 + y^2} = \rho \sin \phi,$$

$$\rho = \sqrt{x^2 + y^2 + z^2},$$

$$x = r \cos \theta = \rho \sin \phi \cos \theta,$$

$$y = r \sin \theta = \rho \sin \phi \sin \theta,$$

$$z = \rho \cos \phi,$$

$$dV = dz \, dy \, dx$$

$$= r \, dz \, dr \, d\theta$$

$$= \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta.$$

