Vector Calculus Formulas

Fundamental theorems (main result) Here, \(F(x, y, z) = P(x, y, z)i + Q(x, y, z)j + R(x, y, z)k\).

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
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| FT of Line Integrals: | If \(F = \nabla f\), and the curve \(C\) has endpoints \(A\) and \(B\), then \[
\int_C F \cdot dr = f(B) - f(A).
\] |
| Green’s Theorem: | \[
\iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \oint_C F \cdot dr \quad \text{(circulation-curl form)}
\] |
| Stokes’ Theorem: | \[
\iint_S \nabla \times F \cdot \mathbf{n} \, d\sigma = \oint_C F \cdot dr, \quad \text{where } C \text{ is the edge curve of } S
\] |
| Green’s Theorem: | \[
\iint_D F \cdot n \, dA = \oint_C F \cdot dr \quad \text{(flux-divergence form)}
\] |
| Divergence Theorem: | \[
\iiint_D \nabla \cdot F \, dV = \iint_S F \cdot n \, d\sigma
\] |

Differential elements

Along a parametrized curve \(r(t), t \in [a, b]\), we have \(ds = \left| \frac{dr}{dt} \right| dt\).

Along a parametrized surface \(r(u, v), (u, v) \in D\), we have \(d\sigma = \left| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right| du \, dv\).

Curl and divergence

For a continuously differentiable 3D vector field \(F(x, y, z) = P(x, y, z)i + Q(x, y, z)j + R(x, y, z)k\),

\[
\text{curl } F = \nabla \times F := \left( \frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y} \right)i + \left( \frac{\partial R}{\partial x} - \frac{\partial P}{\partial z} \right)j + \left( \frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right)k.
\]

\[
\text{div } F = \nabla \cdot F := \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot (P, Q, R) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}.
\]

Other 3D spatial coordinates

\[
r = \sqrt{x^2 + y^2} = \rho \sin \phi,
\]
\[
\rho = \sqrt{x^2 + y^2 + z^2},
\]
\[
x = r \cos \theta = \rho \sin \phi \cos \theta,
\]
\[
y = r \sin \theta = \rho \sin \phi \sin \theta,
\]
\[
z = \rho \cos \phi,
\]
\[
dV = dz \, dy \, dx = r \, dz \, dr \, d\theta = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta.
\]