Reading Questions for Boyce and DiPrima, Section 7.4

[Submit your responses by 3 am, Fri., Apr. 12, using the webform below.]

1. Much of the theory of solving linear 1st-order systems of DEs

\[ \frac{d}{dt}x(t) = P(t)x + g(t), \]

should sound familiar. In particular, one solves the complementary (homogeneous) problem

\[ \frac{d}{dt}x(t) = P(t)x \]

first and, to do that, you seek \( n \) (I’m assuming \( P \) is an \( n \)-by-\( n \) matrix) linearly independent (vector function) solutions \( \{x^{(1)}, \ldots, x^{(n)}\} \) to serve as a fundamental set of solutions, and which serve as the building blocks (via linear combinations) for all solutions of the homogeneous problem. If you have \( n \) such vector function solutions, one can check linear independence via a Wronskian, as in Chapter 3. But the Wronskian is built a little differently. Consider the three vector functions

\[
x^{(1)}(t) = e^{-t} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \quad x^{(2)}(t) = e^t \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}, \quad \text{and} \quad x^{(3)}(t) = e^{2t} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix},
\]

which all solve the 1st order system

\[
\frac{d}{dt}x = \begin{bmatrix} 11/9 & -2/9 & 8/9 \\ -2/9 & 2/9 & 10/9 \\ 8/9 & 10/9 & 5/9 \end{bmatrix} x.
\]

What does the Wronskian \( W[x^{(1)}, x^{(2)}, x^{(3)}](t) \) look like? Is there a situation in which the Wronskians of both Chapters 4 and 7 would agree?

2. Identify one item (a concept, a step in an example, a statement, etc.) from this reading assignment you found difficult or confusing.