Reading Questions for Boyce and DiPrima, Section 1.1

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1. The authors make the statement, "It is often assumed that the drag (force due to air resistance) is proportional to the velocity." Why should such a force then take the form \( pv \)? Does this assumption about drag force seem reasonable to you? Why or why not?

2. Differential equations, as you will discover, is a subject for which the form of an equation matters a great deal. According to the authors, "Direction fields are valuable tools in studying the solutions of DEs of the form \( y' = f(t, y) \)," a first order ODE. They go on to say that one does not need to solve this DE in order to construct a direction field (which can offer valuable insight to the structure of solutions). Make sure you understand just how a direction field is constructed (i.e., so that you could do it by hand, given enough time).

   (a) Why is a direction field a concept associated only with \textit{first order} DEs, in the form above? Why, say, are we unable to use a direction field to investigate the behavior of the 2nd order DE \( y'' + 2y' - ty = 4 \)?

   (b) The text points out that Equation (5) \( v' = 9.8 - (0.2)v \) is one in the form \( v' = f(v) \) (i.e., the function \( f \) is of the dependent variable \( v \) alone and not of the independent variable \( t \)). Such a first order DE is said to be \textbf{autonomous}. Why should it be a good deal easier to construct (again, by hand) a direction field for an autonomous first order DE as opposed to a non-autonomous one \( v' = f(t, v) \)?

3. The term \( dp/dt \) in Equation (8) is a rate of change for a certain population of mice. What are the units of measurement for \( dp/dt \)? The other two terms, \( (0.5)p \) and \( (450) \), must be rates as well, measured in the same units. Explain the role each of these terms plays, and why they have opposing signs. Can you identify ways in which this model seems to be lacking in its description of the mice population? (Try to think of another answer besides ones given in the text.)

4. Can Newton’s second law (Equation (1)) itself be thought of as a DE? If so, write it as one, indicating its order.

5. Think of some of the different ways you use the term “model”. In what way do the differential equations from examples in the section fit with your idea of models?

6. Identify one item (a concept, a step in an example, a statement, etc.) from this reading assignment you found difficult or confusing.