Peer Questions for Section 8.4

In your group (minimum of two people per), discuss your responses to the following questions. Rotate (again) the role of "group scribe", a person who should submit your group’s responses, using the web form below, by 5 pm, Tues., Oct. 15.

The most important portions of this section are on the Ratio Test and Root Test, pp. 460–462. But you must know some things from pp. 454-459, and here are the main things from those prior pages:

- You must know what it means for a series to converge absolutely, and the theorem on p. 459. As one application of this result, the knowledge that the \( p \)-series
  \[
  \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots
  \]
  (with \( p = 2 \)) converges tells us that each of the following variants
  \[
  \sum_{n=1}^{\infty} (-1)^{n+1} n^2 = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots, \quad \text{(every other term is negative)}
  
  1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + \cdots, \quad \text{(every third term is negative)}
  
  1 + \frac{1}{4} - \frac{1}{9} - \frac{1}{16} - \frac{1}{25} - \frac{1}{36} + \cdots, \quad \text{(all but first two terms made negative)}
  
  1 - \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + \cdots, \quad \text{(random terms made negative)}
  \]
  also converges. The root and ratio tests assume that you are interested in knowing if the series \( \sum a_n \) converges absolutely, which explains the absolute value symbols around the terms.

- You must know that, while absolute convergence implies convergence, the converse of this result is untrue. The prototypical example here is the alternating harmonic series (Example 1 in the section)
  \[
  1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots,
  \]
  which converges (to the value \( \ln 2 \), in fact), but does not converge absolutely.

There are no peer questions for this section.