Peer Questions for Section 8.1

In your group (minimum of two people per), discuss your responses to the following questions. Rotate (again) the role of “group scribe”, a person who should submit your group’s responses, using the web form below, by 5 pm, Wed., Oct. 2.

1. Fill in the blanks:

   A sequence is a function whose domain is _________________.
   The only limit it makes sense to consider for a given sequence $a_n$ is the limit as ____________.

2. The section is filled with definitions and theorems about sequences. Since sequences are really functions, these simply serve as reminders of definitions/theorems you encountered early on in your study of calculus. Your task here is to establish a mapping between numbered boxes in Section 8.1 and those in Chapter 1. For instance, Box 1 on p. 427 defines the terms convergent and divergent in regards to sequences, and establishes notation for convergent sequences. The (closest) match for this in Chapter 1 is Box 3 on p. 59. Complete the associations for other boxes in the section.

<table>
<thead>
<tr>
<th>Section 8.1 Reference</th>
<th>Chapter 1 Section No.</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box 1</td>
<td>1.6</td>
<td>Box 3</td>
</tr>
<tr>
<td>Box 2</td>
<td>(a) ___________</td>
<td>(b) _________</td>
</tr>
<tr>
<td>Box 5</td>
<td>(c) ___________</td>
<td>(d) _________</td>
</tr>
<tr>
<td>Limit Laws (top, p. 429)</td>
<td>(e) ___________</td>
<td>(f) _________</td>
</tr>
<tr>
<td>Squeeze Thm. (middle, p. 429)</td>
<td>(g) ___________</td>
<td>(h) _________</td>
</tr>
<tr>
<td>Continuity &amp; Convergence Thm. (p. 430)</td>
<td>(i) ___________</td>
<td>(j) _________</td>
</tr>
</tbody>
</table>

3. If sequences are really just functions of a special type, and if most of the ideas and results for sequences are applications of ideas/results we knew to hold generally for functions, one might wonder, “Is there some benefit in studying them?” In the days to come, I hope you will see several satisfactory answers to this question. But I wish to suggest, even at the outset, that the question is wrong-headed. Through much of calculus to this point, we have done problems which presumed we had some base formula $f(x)$ for a desired quantity (perhaps the depth of a pond at position $x$, or the velocity of a car at time $x$, etc.) which yields the value at any $x$ we wish. Real-life problems do not come with such formulas. If we obtain numbers at all, they come to us as data (perhaps as two sequences, one providing the $x$-coordinate and the other sequence giving the corresponding $y$-coordinates) and, if we obtain a formula at any stage, it serves as a mathematical model for the data.

Think of an example from your life where you pay attention to a sequence of numbers.
4. The last problem is not meant to disparage the topics from calculus—those requiring a
mathematical model—prior to Chapter 8. Quite the contrary, Chapters 1–7 have provided
us with tools we might use to answer questions about sequences. Example 4 provides one
instance of this; compare it with Example 5 on p. 62.

Example 5 (back in Section 8.1) provides another instance. It rests on the fact that the graph
of \( f(x) = \frac{\ln x}{x} \) (with domain \( 0 < x < \infty \)) passes through the points of the graph of the
sequence \( a_n = \frac{\ln n}{n} \), and hence serves as a mathematical model for the behavior exhibited
by that sequence. To get the limit of the sequence, then, we can take the limit \( \lim_{x \to \infty} \frac{\ln x}{x} \),
which is easily done using L'Hospital's Rule).

Indicate whether the statement is true or false.

(a) ______ Let us suppose \( (a_n)_{n=1}^{\infty} \) is some given sequence, and that \( f(x) \) is a function
defined for all \( 0 \leq x \leq \infty \) whose graph passes through all points on the graph of the
sequence. If \( \lim_{x \to \infty} f(x) = L \), then \( \lim_{n \to \infty} a_n = L \), too.

(b) ______ Given any two functions \( f \) and \( g \), defined on \( 0 \leq x \leq \infty \), where each passes
through the points on the graph of a given sequence \( (a_n)_{n=1}^{\infty} \), both \( f \) and \( g \) will have the
same limit as \( x \to \infty \).

(c) ______ If we have a formula involving \( n \) for a sequence (like one of those for \( a_n \) in the
bottom middle of p. 425), we can always find \( \lim_{n \to \infty} a_n \) by first converting our formula in
\( n \) to a formula in \( x \), considering the domain for this latter expression to be expanded to
all of \( 0 \leq x < \infty \), and evaluating the limit of this latter function as \( x \to \infty \).

5. For this particular evening’s questions, send me your answers individually ahead of time
using the web form. (Make sure I receive them before the start of next class period, Wed.,
Oct. 2.) Once you have discussed them in group on that day, follow the usual procedure,
having a scribe send me the group’s answers.

6. Identify one item (a concept, a step in an example, a statement, etc.) from this reading
assignment you found difficult or confusing.