MATH 162: Calculus II, Sections B/C

Information for Exam 4
to take place on May 4, 2007

General Information. The exam focuses on material covered in frameworks for the dates Apr. 4–May 1. This corresponds to Sections 12.7 (optimization of functions of 2 variables, both without and with constraints) and 13.1–13.7 (multiple integration) from the text.

As you should be aware from homework, the integration techniques from Chapter 7 often come in handy when calculating the iterated integrals of Chapter 13. You will, indeed, need to demonstrate your ability to calculate a variety of types of integrals for the coming final exam. However, as they pertain to Exam 4, you should focus on proficiency in using the *substitution rule* first learned in MATH 161 (Sections 5.5–5.6).

Formulas. Once again, none will be provided. Some useful formulas you might look at closely for the exam include

1. Given a function of two variables $f(x,y)$,
$$D(x,y) \;:=\; \begin{vmatrix} f_{xx}(x,y) & f_{xy}(x,y) \\ f_{xy}(x,y) & f_{yy}(x,y) \end{vmatrix} \;=\; f_{xx}f_{yy} - (f_{xy})^2.$$

2. The **area** of a bounded region R of the plane is given by
$$\text{Area}(R) \;=\; \iint_R 1\,dA.$$
The **volume** of a bounded region D of 3-dimensional space is given by
$$\text{Vol}(D) \;=\; \iiint_D 1\,dV.$$

3. The **average value** of a function $f(x,y)$ of two variables over a bounded region R of the plane is given by
$$\frac{1}{\text{Area}(R)} \iint\limits_R f(x,y)\,dA.$$
The average value of a function $f(x,y,z)$ of three variables over a bounded region D of 3-dimensional space is given by
$$\frac{1}{\text{Vol}(D)} \iiint\limits_D f(x,y,z)\,dV.$$

4. If $\rho(x,y,z)$ gives the mass density (mass per unit volume) in a distributed body residing in a region D of 3-dimensional space, then the total **mass** M of this body is
$$M \;=\; \iiint\limits_D \rho(x,y,z)\,dV.$$

The **first moments** about $x = 0$, $y = 0$ and $z = 0$ are given by

$$M_{yz} := \iiint\limits_D x\rho(x,y,z)\,dV, \quad M_{xz} := \iiint\limits_D y\rho(x,y,z)\,dV, \quad \text{and} \quad M_{xy} := \iiint\limits_D z\rho(x,y,z)\,dV,$$

respectively. The **center of mass** is located at $(\bar{x}, \bar{y}, \bar{z})$, where $\bar{x} = M_{yz}/M$, $\bar{y} = M_{xz}/M$, and $\bar{z} = M_{xy}/M$.

5. **Relationships between coordinate systems** are most easily remembered from the appropriate pictures; see those provided on the Framework for Apr. 30. The following formulas are among thoes which follow from those pictures:

   (a) $r^2 = x^2 + y^2, \qquad \tan\theta = y/x$

   (b) $\rho^2 = x^2 + y^2 + z^2 = r^2 + z^2$

   (c) $r = \rho\sin\phi, \qquad z = \rho\cos\phi$

   (d) $x = r\cos\theta = \rho\sin\phi\cos\theta, \qquad y = r\sin\theta = \rho\sin\phi\sin\theta$

6. **Volume elements** in the various coordinates are given by

$$dV = \begin{cases} dz\,dy\,dx & \text{in rectangular coordinates} \\ r\,dz\,dr\,d\theta & \text{in cylindrical coordinates} \\ \rho^2\sin\phi\,d\rho\,d\phi\,d\theta & \text{in spherical coordinates} \end{cases}$$

or any permutation thereof.