MATH 162: Calculus II, Sections B/C

Information for Exam 2
to take place on Mar. 7, 2007

General Information.

1. The exam focuses on material from the frameworks for the dates Feb. 13–Mar. 1, 2007. This roughly corresponds to material found in Sections 8.2–8.8, 10.1 and 12.1–12.3, but the frameworks are a much better indicator of what was actually covered from these sections of the text.

2. Homework problems are good indicators of the level of difficulty for problems on the exam. Additional sample problems for *series* appear beginning on p. 573. While it may be helpful to look over some from numbers 19–68 of these additional "Practice Exercises," keep in mind that some require "convergence tests" which we have not learned in class.

3. In addition to *problems*, the exam may include questions which ask you to explain concepts. An example of such a question: "State precisely what one means by the phrase, 'a series converges'."

4. Among all tests for convergence and divergence of series, we have emphasized only the n^{th}-term, the absolute convergence and ratio tests. Be able to state precisely what conditions are required to use each test, and what those conditions allow you to conclude.

5. Several other series have been analyzed without the use of a formal test: geometric series, p-series (alternating and straight), and telescoping series. You should be able to recognize these when they appear and draw the appropriate conclusions.

   We were also able to derive series expressions (and intervals of convergence) using substitution and the theorems about term-by-term integration/differentiation.

6. Some of you may have graphing calculators which can produce graphs for functions $z = f(x, y)$. While you will not be required to set such calculators aside, expect problems to be stated and graded in such a way that the mathematical analysis takes precedence over button-punching skills. A possible problem in this vein might be one in which you are provided a contour plot (a graph that includes several level curves, like what you produced for Problem 22b, p. 710) for $f(x, y)$ and are asked to sketch the graph (surface) $z = f(x, y)$, despite not being given a formula for $f(x, y)$.

7. Certain MacLaurin series, like those for $\sin x$ and e^x, will be provided if needed, except in cases where the problem is actually *about* deriving a Taylor series about $x = a$. For such problems remember that there are sometimes easier ways to come up with the answer than the tedious (but also valid) method of

Finding $f(a)$.
Finding a formula for $f'(x)$. Then evaluating $f'(a)$.
Finding a formula for $f''(x)$. Then evaluating $f''(a)$.
$\vdots$
Finding a formula for $f^{(n)}(x)$. Then evaluating $f^{(n)}(a)$.
$\vdots$

For instance, since we already know the MacLaurin series for

$$\cos x \;=\; 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots ,$$

we may obtain the MacLaurin series for part 3(b) on Friday's handout via substitution:

$$\begin{aligned}
\frac{cos(x^2) - 1}{x^4} &= \frac{1}{x^4}\left[\left(1 - \frac{(x^2)^2}{2!} + \frac{(x^2)^4}{4!} - \frac{(x^2)^6}{6!} + \cdots\right) - 1\right] \\
&= \frac{1}{x^4}\left[\left(1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots\right) - 1\right] \\
&= \frac{1}{x^4}\left(-\frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots\right) \\
&= -\frac{1}{2!} + \frac{x^4}{4!} - \frac{x^8}{6!} + \cdots .
\end{aligned}$$

(Notice that this series produces the correct value at $x = 0$ as well, as defined for the given f on that handout.)