MATH 143: Introduction to Probability and Statistics

Worksheet 9 for Thurs., Dec. 10: What procedure?

For each numbered problem, identify (if possible) the following:

(a) the variable(s) and variable type(s) of interest.

(b) the type of inference procedure (1-sample z, 1-sample t, 1-proportion, 1-way ANOVA, 2-sample t, 2-way ANOVA, simple linear regression, multiple regression, etc.)

(c) Is an hypothesis test or a confidence interval what is called for?

(d) If an hypothesis test were performed using this data,

 (i) what would be appropriate hypotheses?

 (ii) what type of test statistic? (z? t? χ^2? Something else?)

(e) Is it possible to use this data to construct a confidence interval? If so,

 (i) for what population parameter (or combination of population parameters)?

 (ii) what type of critical value would be used?

 (iii) what formula for the appropriate standard error?

1. We have a sample of 35 frisbies for which the weight (in ounces) and distance (in feet) of flight when thrown by a mechanical arm are recorded. We wish to know a likely range of numbers that represent how that distance of flight changes when the weight of the frisbee is increased by 1 ounce.

 (a) weight (quantitative) and distance traveled (quantitative)

 (b) simple linear regression

 (c) confidence interval

 (d) (i) $\mathbf{H}_0:\ \beta = 0$, $\mathbf{H}_a:\ \beta \neq 0$

  (ii) t-statistic ($df = 33$)

 (e) Absent the fact that we have no distance data, yes it is possible.

  (i) β, the slope of the (true) regression line between distance (response variable) and weight (explanatory var.)

  (ii) t critical value ($df = 33$)

  (iii) $\dfrac{s}{\sqrt{\sum(x-\bar{x})^2}}$ (one we never used, and you need not know)

2. *Sports Illustrated* magazine surveyed a random sample of 757 Division I college athletes in 36 sports. One question asked was “Have you ever received preferential treatment from a professor because of your status as an athlete?” Of the athletes polled, 225 said “Yes.” What value(s) do we think likely for the true percentage of athletes who believe they have received this kind of preferential treatment?

(a) There are several possible answers here. But, if we consider variables measured on the actual respondents, it is this: ‘‘whether or not preferential treatment was sensed", a categorical variable, with values ‘‘yes" and ‘‘no".

(b) 1-proportion

(c) confidence interval

(d) (i) It is difficult to state null and alternative hypotheses without further information. We do not have an indication in the problem of any widely-accepted value for this parameter.

(ii) z-statistic

(e) Yes, it is possible using just the information given.

(i) p, the true proportion of Division I athletes who feel they have received preferential treatment.

(ii) z critical value

(iii) $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, with $\hat{p} = \frac{225}{757}$ (assuming a large-sample method)

3. When the new euro coins were introduced throughout Europe in 2002, curious people tried all sorts of things. Two Polish mathematicians spun a Belgian euro (one side of the coin has a different design for each country) 250 times. They got 140 heads. Newspapers reported this result widely. Is it significant evidence that the coin is not balanced when spun?

(a) the outcome of a spin (categorical, with value ‘‘head" or ‘‘tail")

(b) 1-proportion

(c) hypothesis test

(d) (i) $\mathbf{H}_0$: $p = \frac{1}{2}$, $\mathbf{H}_a$: $p \neq \frac{1}{2}$

(ii) It is a z test statistic.

(e) Yes, it is possible.

(i) p, the true proportion of ‘‘heads" that occur in the long run over many spins of a euro coin.

(ii) z critical value

(iii) $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, with $\hat{p} = \frac{140}{250}$ (assuming a large-sample method; Note that this is not the same standard error one would use for the hypothesis test of part (d).)

4. The corn from three different seed types is planted in an experiment involving 5 different types of fertilizers. We wish to know if certain pairings of seed type–fertilizer type grow, on average, higher than others.

(a) seed type (categorical), fertilizer type (categorical), height of plant after a certain period of time (quantitative)

(b) 2-way ANOVA

(c) hypothesis test (that's all that is available in ANOVA)

(d) (i) $\mathbf{H}_0$: The mean heights (μ_i's) for all groups are equal.
$\mathbf{H}_a$: At least one mean height (μ_i's) is different than others.

(ii) F-statistic

(e) Not without isolating our focus down to two specific groups.

5. The amount of lead in a certain type of soil, when released by a standard extraction method, averages 86 parts per million (ppm). A new extraction method was tried, with researchers wondering if this new approach would result in a significant difference in the mean amount of extracted lead. Forty one specimens were obtained, with a mean of 83 ppm lead and a s.d. of 10 ppm.

(a) amount of lead in ppm (quantitative)

(b) 1-sample t

(c) hypothesis test

(d) (i) $\mathbf{H}_0$: $\mu = 86$, $\mathbf{H}_a$: $\mu \neq 86$

(ii) t-statistic

(e) Yes, it is possible.

(i) μ, the mean amount of lead under this new extraction method.

(ii) t critical value, with $df = 40$

(iii) $s/\sqrt{n} = 10/\sqrt{41}$

6. Many low- and middle-income families do not save enough for their retirement. It would be to their advantage to contribute to an individual retirement account (IRA), which allows money to be invested for retirement without paying taxes on it now. Would more families contribute to an IRA if the money they invest were matched by their employer? In an experiment on this question, the tax firm *H&R Block* offered to partly match IRA contributions of families with incomes below $40,000. In all, 1681 married taxpayers were assigned at random to the control group (no match), 1780 to a 20% match, and 1831 to a 50% match. All were offered the opportunity to open an IRA. The study found that 49 married taxpayers in the control group, 240 in the 20% group, and 456 in the 50% group opened IRAs. Is there strong evidence that people in these groups decide differently about opening IRA accounts?

(a) which group (categorical, with values zero-matching, 20%-matching, and 50%-matching), and whether an IRA was opened (categorical)

(b) chi-square

(c) hypothesis test (all that is possible with chi-square)

(d) (i) $\mathbf{H}_0$: the percentage of people opening IRAs is the same in all groups
$\mathbf{H}_a$: the percentages are not the same in every group

(ii) χ^2

(e) No, it is not possible, unless we shed one of our groups (see Problem 7).

7. Consider the *H&R Block* study described above. We wish to estimate the difference between the percentages of people opening an IRA when offered a 50% match vs. those offered a 20% match. (Note: We are ignoring the control group here.)

   (a) same variables as for Problem 6

   (b) 2-proportion

   (c) confidence interval

   (d) (i) $\mathbf{H}_0$: $p_{50\%} - p_{20\%} = 0$, $\mathbf{H}_a$: $p_{50\%} - p_{20\%} > 0$
   (1-sided alternative seems appropriate here)

   (ii) z-statistic

   (e) (i) the difference of proportions $p_{50\%} - p_{20\%}$

   (ii) z critical value

   (iii) $\sqrt{\frac{p_{50\%}(1-p_{50\%})}{n_{50\%}} + \frac{p_{20\%}(1-p_{20\%})}{n_{20\%}}}$ (assuming large-sample procedure)

8. Consider the same *H&R Block* study. Each taxpayer who opened an IRA decided how much to contribute. Those in the control group contributed \$1549 on average, with standard deviation \$1652. Those who were offered a 20% match contributed an average of \$1723 with s.d. \$1332, and those offered a 50% match contributed an average of \$1742 with s.d. \$1174. Is this strong evidence to conclude that people with these three different types of incentives for contributing to an IRA will make different decisions about how much to contribute?

   (a) which group (categorical, with values zero-matching, 20%-matching, and 50%-matching), and amount contributed (quantitative)

   (b) 1-way ANOVA

   (c) hypothesis test (the only option from 1-way ANOVA)

   (d) (i) $\mathbf{H}_0$: average contributions for people in all 3 groups are the same
   $\mathbf{H}_a$: at least one group average is different from others

   (ii) F-statistic

   (e) No, nothing for all three group means at once; only *pairwise comparisons* may be made.

9. With the data of the previous problem, estimate the difference in the average contributions among people who open an IRA under a 50% match program and those who do so under a 20% match program.

(a) which group (categorical, with values 20%-matching, and 50%-matching), and amount contributed (quantitative)

(b) 2-sample t

(c) confidence interval

(d) (i) $\mathbf{H}_0$: $\mu_{50\%} - \mu_{20\%} = 0$, $\mathbf{H}_a$: $\mu_{50\%} - \mu_{20\%} \neq 0$
(I see less of a reason here for a 1-sided alternative)

(ii) t-statistic ($df = 1779$)

(e) Yes, it is possible.

(i) the difference of means $\mu_{50\%} - \mu_{20\%}$

(ii) t critical value ($df = 1779$)

(iii) $\sqrt{\dfrac{s^2_{50\%}}{n_{50\%}} + \dfrac{s^2_{20\%}}{n_{20\%}}}$

List the standard assumptions/conditions which validate

I. inferences on a single (population) mean.

See the yellow box on p. 434. There is a description of the meaning of **robust** on p. 447, and some rules of thumb about sample sizes are given on p. 448.

II. inferences on the difference of two (population) means.

See the yellow box on p. 462. See what is said about **robustness** on the top of p. 473.

III. inferences on a single proportion.

See p. 495. Also, at the bottom of yellow boxes on p. 497, p. 500 and p. 504 are given rules of thumb concerning counts of successes to require when doing *large-sample confidence intervals*, *plus-four CIs*, and *hypothesis tests*.

IV. inferences on the difference of two proportions.

Like with other inference procedures involving two or more groups/populations, we need to be able to consider the samples from these groups as *indepedent SRSs*. For rules of thumb concerning counts of success and failures, see the yellow boxes on p. 515, p. 518 and p. 521.

V. inferences on the value of a model parameter in simple or multiple regression

The assumptions for both may be summed up as: linearity, normality, constant variance, and independence. See p. 583 for better descriptions of these in the case of *simple linear regression*.

VI. inferences about whether there is some relationship between a collection of potential explanatory quantitative variables and some quantitative response variable.

I don't recall what I had in mind when I wrote this one. So far as I know, the assumptions are the same as in Question V.

VII. our methods for determining if there is some relationship between two categorical variables.

Like with other inference procedures involving two or more groups/populations, we need to be able to consider the samples from these groups as *indepedent SRSs*. For rules of thumb concerning expected counts, see p. 560.

VIII. our methods for determining if there is some difference in population means amongst several different populations.

See p. 633. For a reminder of the rule of thumb for checking the assumption about population/group standard deviations, see p. 634.

What does it mean to say a certain inference procedure is *robust* against a particular assumption? Go back through the list of conditions you made for the previous problem. Circle those assumptions against which the corresponding inference procedure is robust.