

PROBLEM OF THE WEEK

Welcome back everyone! Below is the first problem of the new year.

I tried to find out over the summer just how long we have had a problem of the week. It appears to be at least 25 years, but no one is quite sure. Since I couldn't even determine how long it has been going, there was no way for me to figure out how many problems there have been, but it seems quite certain to be well over 500. You can join a long tradition of Calvin students by submitting a solution to a problem.

I've decided to start numbering problems consecutively beginning in January, 2000. That makes this problem 37 (at least approximately) of the 2000's.

I encountered this problem at MathFest this summer. This result was used to perform several mathematical card tricks. No proof was given in the talk, so I played around with it myself. I hope you enjoy it, too.

- 37.** Consider a finite sequence of distinct integers. A subsequence is a sequence formed by deleting some items from the original sequence without disturbing their relative ordering. A subsequence is called monotone if it is either increasing (each term is larger than the one before it) or decreasing (each term is smaller than the one before it). For example, if the sequence is 4, 6, 3, 5, 7, 1, 2, 9, 8, 10, then 4, 6, 8, 10 is a monotone (increasing) subsequence of length 4 and 6, 5, 2 is a monotone (decreasing) subsequence of length 3.
- Find a sequence of 9 distinct integers that has no monotone subsequence of length 4.
 - Show that every such sequence of length 10 has a monotone subsequence of length 4.
 - Generalize this result. (How long must the sequence be to guarantee a monotone subsequence of length n ?)

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- SOLUTIONS** to this problem are due on **Thursday, September 13**. Solutions should be turned in to Professor Pruim (NH 284). Be sure to include your name(s) on your paper.
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PROBLEM OF THE WEEK

Just a reminder: the problem of the week is distributed by email to those subscribed to math-news and is also available on the web. See the Department Web Site for details.

This week's problem was contributed by **Professor Talsma**.

- 38.** The number $1000000!$ (one million factorial) ends with lots of zeros. How many zeros? What is the rightmost non-zero digit?

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3. **SOLUTIONS** to this problem are due on **Thursday, September 20**. Solutions should be turned in to Professor Pruim (NH 284). Be sure to include your name(s) on your paper.
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PROBLEM OF THE WEEK

This week's problem was contributed by Professor Ankney.

39. "I hear some youngsters playing in the backyard," said Jones, a mathematics student. "Are they all yours?"

"No," explained Professor Smith, "My children are playing with friends from three other families in the neighborhood, although our family happens to be the largest. The Browns have a smaller number of children, the Greens have a still smaller number, and the Blacks have the smallest of all."

"How many children are there all together?" asked Jones.

"Let me put it this way," said Professor Smith. "There are fewer than 18 children, and the product of the numbers in the four families happens to be my house number, which you saw when you arrived."

Jones took a notebook and pencil from his pocket and started scribbling. A moment later he looked up and said, "I need more information. Is there more than one child in the Black family?"

As soon as Professor Smith replied, Jones smiled and correctly stated the number of children in each family.

Knowing the house number and whether the Blacks had more than one child, Jones found the problem trivial. It is a remarkable fact, however, that the number of children in each family can be determined solely on the basis of the information given above!

Determine the number of children in each family.

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3. SOLUTIONS to this problem are due on **Thursday, September 27**. Solutions should be turned in to Professor Pruim (NH 284). Be sure to include your name(s) on your paper.
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PROBLEM OF THE WEEK

There are a number of problems that involve weighing items on scales. Here are two. The first one is a new one (to me) that I came across lately. The other an old favorite. Feel free to submit a solution to either or both. Enjoy.

40. Suppose you are given two red balls, two blue balls, and two white balls, together with the information that one ball of each color has weight A and one ball of each color has weight B , with A not equal to B . Using only an equal-arm balance, what is the smallest number of weighings that suffices to learn the weights of all of the balls?
41. Weird Wizard Wally offers you the following challenge. Wally will provide you with an equal-arm balance and n visually indistinguishable coins. Exactly one of the n coins is counterfeit and either slightly heavier or slightly lighter than the remaining coins, but you are not told which. You are allowed only three weighings, after which you must identify the counterfeit coin and whether it is too heavy or too light. If you are correct, you keep all the coins ($n - 1$ valuable coins and one worthless coin). If you are incorrect, you get nothing.
- How many coins should you choose? (How large should n be?)

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3. SOLUTIONS to this problem are due on **Thursday, October 4**. Solutions should be turned in to Professor Pruim (NH 284). Be sure to include your name(s) on your paper.
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PROBLEM OF THE WEEK

42. Call a set of points in the plane *nice* if

- there are at least three points in the set,
- no 3 are collinear, and
- for any 3 points in the set, the center of the circle containing these points is also in the set.

a) Can a nice set be finite?

b) Can a nice set be countable?

c) Notice that this problem assumes that for any three non-collinear points there is a unique circle that goes through all three. Prove this too.

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PROBLEM OF THE WEEK

- 43.** A $2 \times 4 \times 6$ inch rectangular solid has the property the its volume is 48 cubic inches and the total length of its 12 edges is 48 inches. Are there any other triples of integers like $(2, 4, 6)$ for which the volume and the total edge length of the corresponding rectangular solid are numerically equal? Find all possibilities and prove that there are no others.

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3. **SOLUTIONS** to this problem are due on **Thursday, October 18**. Solutions should be turned in to Professor Pruim (NH 284). Be sure to include your name(s) on your paper.
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PROBLEM OF THE WEEK

This problem is related to one that appeared last year. (Last year's problem was to show the existence of monochrome equilateral triangles.)

44. Let every point in the plane be colored either red or blue. Call a rectangle *monochrome* if each of its vertices is the same color.
- Show that no matter how the colors are assigned to the points in the plane, there will be a monochrome rectangle.
 - Can you improve on this by (i) increasing the number of colors, or (ii) requiring that the rectangles be squares?

You are invited to submit a solution even if you only get one part of the problem.

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PROBLEM OF THE WEEK

This is a different sort of problem from most problems of the week. It is a modeling problem. The answer you get may depend on just how you choose to model the situation.

- 45.** As I was bicycling home in the rain one day last week I mused about the following question: How fast should I ride and what difference does it make? Clearly as I rode faster I got wet faster (by plowing into the rain at a faster rate as I moved forward). On the other hand, I was going to spend less time in the rain. So here is a specific question one could ask:

Assume a constant rate of rainfall. I live 2 miles from campus. Will I get wetter if I ride at 10 mph or at 20 mph? How much wetter? Or doesn't it matter at all?

Feel free to generalize the problem in any way you like. Your solution should indicate any assumptions you make.

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3. **SOLUTIONS** to this problem are due on **November 1**. Solutions should be turned in to Professor Pruim (NH 284). Be sure to include your name(s) on your paper.
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PROBLEM OF THE WEEK

Approximately weekly I receive from Stan Wagon at Macalaster College an email with the current Macalaster College Problem of the Week. Occasionally these problems get elevated to the status of Calvin College Problem of the Week. Here is one such problem (the current problem at Macalaster College):

- 46.** a) A group of 16 people wishes to play 7 rounds of golf in foursomes; that is, each round consists of four groups of four playing the course. They want to arrange things so that each pair plays together at least once in a foursome. Design a scheme to arrange this.
- b) One can generalize this problem: How many rounds are required to allow each pair from n golfers to golf in the same foursome at least once? (See the note below.)

The following note accompanied this problem:

This is a real-world problem suggested by a (non-math) colleague, Mark Davis. Mark conjectures that if the number of golfers is n , divisible by 4, then the number of rounds needed so that everyone plays with everyone else is $(n/2) - 1$. But we have no proof. Naturally, we also wonder if this problem falls within the purview of well-known methods in combinatorial design. Comments welcome.

If you solve the generalized version, I will be sure to pass your solution along to the organizers of the Macalester College Problem of the Week.

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3. **SOLUTIONS** to this problem are due on **November 8**. Solutions should be turned in to Professor Pruim (NH 284). Be sure to include your name(s) on your paper.
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PROBLEM OF THE WEEK

We haven't had a geometry problem in a while, so I guess it is about time.

- 47.** An n -sided equiangular polygon is inscribed in a circle. Show that if n is odd, then the polygon must be regular but that for any even n there is a non-regular, equiangular polygon that can be inscribed in a circle.

As always, partial solutions are welcome.

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3. **SOLUTIONS** to this problem are due on **November 15**. Solutions should be turned in to Professor Pruium (NH 284). Be sure to include your name(s) on your paper.
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PROBLEM OF THE WEEK

Since November 22 is Thanksgiving Day, the deadline for these problems is November 29. Feel free to take them home and try them on your friends and family.

48. Four golfballs can be positioned in such a way that each golfball touches the other 3. This can be done by making a “triangle” of touching golfballs and placing the fourth ball on top of the triangle. Let’s see how you do on these:

- a) How many soup cans can you arrange so that each one touches each of the others?
- b) How many half dollar coins can you arrange so that each one touches each of the others?
- c) How many cigarettes can you arrange so that each one touches each of the others?

Of course you may not bend, cut, melt or otherwise distort the shapes of the objects involved.

Notice that each the objects above is a cylinder. If a cylinder has radius r and length l , call $\frac{l}{r}$ the *stoutness* of the cylinder.

- d) For what range of stoutnesses will your solutions above work?

Submit your best efforts for each of the three problems. Feel free to give credit to Aunt Gladys if she helps you out.

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PROBLEM OF THE WEEK

Welcome to Spring Semester everyone! Now that your first few days of classes are behind you, it's time for another problem of the week. This is a problem from Stan Wagon, often organizer of another long-standing Problem of the Week and author the book *Which Way Did the Bicycle Go?* You can see a picture of him riding an interesting bicycle on the bulletin board between the offices of Professors Brink and Pruiim.

- 49.** Alice: Here's the deal. You give me \$10. Then I will deal four cards (from a regular 52 card deck), chosen randomly, face down. You get to look at #1 first and decide whether to keep it. If not, look at #2 and decide whether to keep that one. If not look at #3, and decide. If you don't take that, then #4 is your choice. If your chosen value is n , I will pay you \$ n . Then we can reshuffle the entire deck, you give me another \$10, and we can play again, and again, and again.

Bob: Hmm...I need a good strategy to beat you at this game, but I think I can do it.

Help Bob out with a strategy that will win. Note that the cards all have face value with the following exceptions: Ace=1, Jack = 11, Queen = 12, and King = 13.

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3. SOLUTIONS to this problem are due on **February 7**. Solutions should be turned in to Professor Pruiim (NH 284). Be sure to include your name(s) on your paper.
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PROBLEM OF THE WEEK

After a bit of a break while we were all busy with the Michigan Undergraduate Mathematics Conference, here is another problem. This one comes (after a bit of modification) from **Professor Hanisch**, who encountered the problem while grading for the Michigan Mathematics Prize competition between semesters.

50. I am thinking of an integer n with $0 \leq n \leq 15$. You get to write 7 yes-or-no questions on a sheet of paper and give them to me. The questions must be independent of each other, their answers, and the order in which they are answered. (So you can't ask a question like "If the answer to the previous question was 'yes', then is the n larger than 10, otherwise is n even?".) Once you give me your 7 questions, I will answer them, but I may lie at most once.

What seven questions can you ask me that will always allow you to determine the number n ?

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3. **SOLUTIONS** to this problem are due on **February 28**. Solutions should be turned in to Professor Pruim (NH 284). Be sure to include your name(s) on your paper.
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PROBLEM OF THE WEEK

A mean problem? Or just another average?

I came across this problem recently on one of my problem sources (the Macalaster Problem of the Week). Looks like a fun problem to think about while you drive somewhere warmer for spring break.

51. Alice: I was driving along a highway recently for one hour at a constant and very special speed.

Bob: What was special about it?

Alice: The number of cars that I passed was the same as the number of cars that passed me!

Bob: It seems as if your speed must have been the median of the speeds of the cars on the road.

Alice: Or was it the mean?

Bob: Those two are often confused. Maybe it's neither? We'll have to think clearly about this.

Help Alice and Bob out? Was Alice's speed the median, the mean, or neither?

Notes: Assume that any car on the road drives at a constant nonzero speed of s miles per hour, where s is a positive integer. And suppose that for each s , the cars driving at speed s are spaced uniformly, with $d(s)$ cars per mile, $d(s)$ being an integer. And because each mile looks the same as any other by the uniformity hypothesis, we can take mean and median to refer to the set of cars in a fixed one-mile segment, the half-open interval $[M, M+1)$, at some instant.

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PROBLEM OF THE WEEK

I had some technical difficulties getting the previous problem printed, and many of you have had your mind on spring break this week, so I'll extend the deadline on problem 51 and make both this problem and that problem due on March 21.

Have a nice spring break; see you in a week or so.

- 52.** In an LCD display some numbers, when viewed upside-down, are images of other numbers. For example, 1995 becomes 5661. The fifth number that can be read upside down is 8, and the 15th is 21, which is 12 when viewed upside-down. What is the millionth number that is meaningful upside-down?

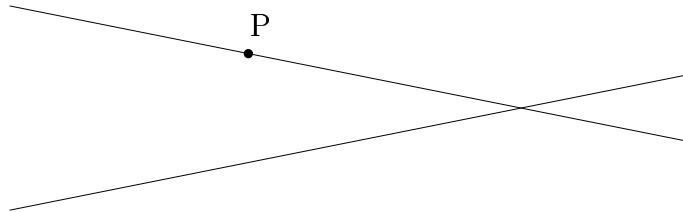
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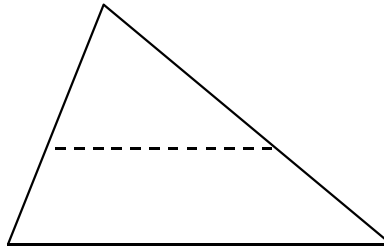
PROBLEM OF THE WEEK

This week we have a couple of geometric constructions to try. The usual rules apply: Constructions must be done using only a (uncalibrated) straight-edge and compass.

- 53.** Given two intersecting lines, and a point P on exactly one of them, construct a circle that is tangent to both lines, with the point of tangency on one line being P .



- 54.** a) Given a triangle with horizontal base, construct a line parallel to the base that divides the triangle into two pieces of equal area.



- b) Can you construct four lines parallel to the base that divide the triangle into 5 pieces of equal area?

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PROBLEM OF THE WEEK

This week's problem comes from my travels to the Joint Mathematics Meetings in California during interim.

- 55.** A certain hotel room lock is opened by scanning a key card. In theory, one enters the room by inserting and removing the key card once. In practice, however, the key card is ambiguously labeled, so that either of two orientations might be the correct orientation of the card. In theory, of course, one could just try one orientation, and if it didn't work try the other. In practice, however, the card reader sometimes fails, so that after trying each orientation once, one may still not have gained access to the room.

A few more details:

- (a) Either of the two orientations is equally likely to be correct.
- (b) The success rate of a correctly oriented card is p with $0 < p < 1$.
- (c) Failure on either side of the card is indistinguishable, so it does not give any information about whether the orientation of the card was correct.
- (d) An attempted scan takes 1 second to succeed or fail; reversing the orientation also takes 1 second.

The last property means that in 3 seconds one could try one orientation 3 times or each orientation once (using the middle second to flip it over). In either case, of course, one might still be standing in the hall and need to decide what to do next.

So what attempt strategy would you use to enter the room? Why?

Feel free to consider fixed values of p ($p = .5$ or $p = .9$, for example) as special cases. What if p is fixed but unknown?

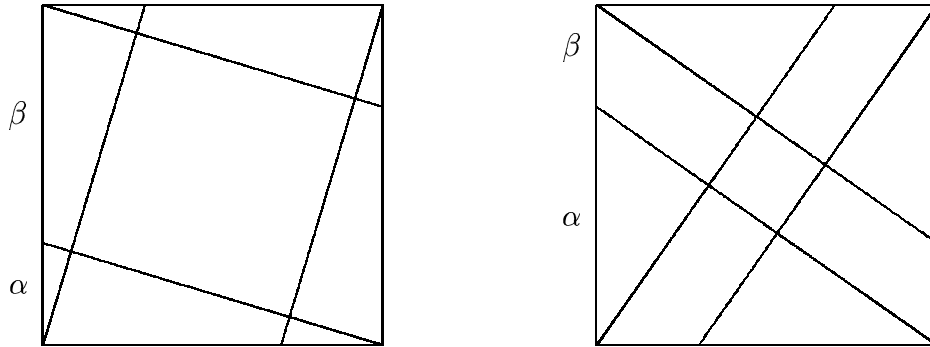
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PROBLEM OF THE WEEK

This week's problem was inspired by the recent colloquium talk by Professor Talsma.

- 56.** Let α and β be positive real numbers summing to 1. On each side of a unit square mark point at a distance of α from one corner in such a way that the lengths of the segments around the square alternate between α and β . Then connect each of these points to a corner of the opposite side of the square to form a right triangle with legs of length α and 1. (See the diagrams below.)



What is the area of the central square formed by these four segments?

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PROBLEM OF THE WEEK

Geometry seems to be a hit of late, so here are two more geometry problems for you to try. Both came from a problem swap I had with Blair Madore when he visited recently. (Swapping problems is the mathematicians version of soldiers swapping war stories.) Enjoy!

- 57.** Consider a set of four distinct points in the plane. For each of the six pairs of points from the set, there is a distance between them. Call the set k -distanced if there are exactly k distinct distances. For example, a long, skinny rectangle is 3-distanced because each distance occurs twice.

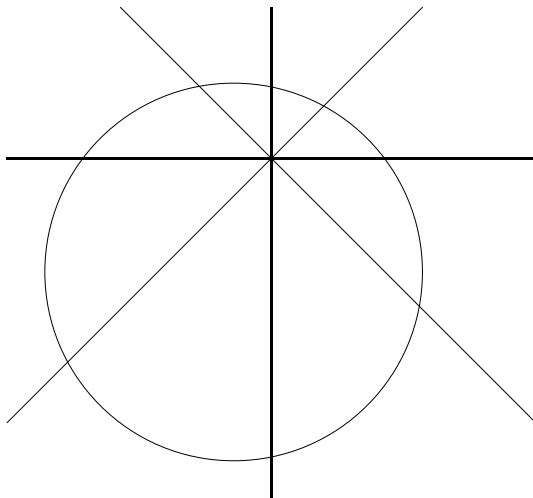
Find as many 2-distanced sets as you can. Can you prove there are no others?

The next problem is really a conjecture. So it might be false. A few of us thought about it for a little while, but then moved on to other problems.

- 58.** At Antonio's Pizza π 's they cut pizzas using an automated pizza slicer that works as follows: The slicer has four straight blades that meet in a common point and extend from that point in both directions. The angle between adjacent blades is 45 degrees. The pizza is slid under the slicer and chopped into 8 slices.

Antonio's pizzas are always perfect circles (of course), and if the pizza is centered under the slicer, the result is 8 slices of equal area. But what if the intersection of the blades is not at the center of the pizza?

Conjecture: The 8 slices can always be partitioned into two sets so that the total area of each set is the same. That is, two people can share the pizza equally without subdividing the slices.



How the Problem of the Week works:

1. **ANY CALVIN STUDENT** is invited to participate in the Problem of the Week on any week. Solutions (or partial solutions) may be submitted by individual students or by groups of students.
2. **COPIES** of the Problem of the Week will be hung on the bulletin board outside the Department office and in various locations around the Department of Mathematics and Statistics. Additional copies are available in one of the boxes outside the office and on the web at <http://www.calvin.edu/~rpruim/pow/>
3. **SOLUTIONS** to this problem are due on **April 18**. Solutions should be turned in to Professor Pruim (NH 284). Be sure to include your name(s) on your paper.
4. **A LIST OF SOLVERS AND EXAMPLE SOLUTIONS** will be posted on the bulletin board outside the Mathematics Department office.

PROBLEM OF THE WEEK

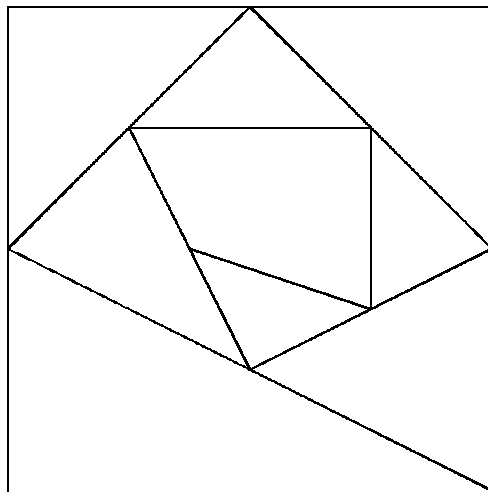
59. Find the smallest positive integer having divisors that end in every decimal digit.

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3. SOLUTIONS to this problem are due on **April 25**. Solutions should be turned in to Professor Pruim (NH 284). Be sure to include your name(s) on your paper.
4. A LIST OF SOLVERS AND EXAMPLE SOLUTIONS will be posted on the bulletin board outside the Mathematics Department office.

PROBLEM OF THE WEEK

- 60.** Let P_0, P_1, P_2, P_3 be the points $(0, 0), (1, 0), (1, 1), (0, 1)$ in the plane going clockwise around a unit square. Now extend the sequence by letting P_{n+4} be the midpoint of P_n and P_{n+1} . The polygonal spiral path $P_0, P_1, P_2, P_3, P_4, P_5, \dots$ approaches a point P in the interior of the original square.
- What are the coordinates of the point P ?
 - Is the path finitely or infinitely long? If it is finite, what is its length?
 - Can you generalize this result to other shapes (starting from non-square quadrilaterals or other polygons)?



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- SOLUTIONS** to this problem are due on **May 2**. Solutions should be turned in to Professor Pruim (NH 284). Be sure to include your name(s) on your paper.
- A LIST OF SOLVERS AND EXAMPLE SOLUTIONS** will be posted on the bulletin board outside the Mathematics Department office.