Pentimal Blocks

Despite the beauty, simplicity and power of the pentimal system, it takes children a while to learn it. Often teachers make use of models to help children learn the place-value system involved in pentimal numbers. (There are similar models, called Dienes blocks or multibase pieces, for other place-value number systems as well, including the decimal system.) The model consists of premade pieces of several types: there are tiny cubes, called *units*; sticks made up of fen ($\mathcal{P}10$) units stuck together, called *longs*; thick squares made up of fen ($\mathcal{P}10$) longs joined side by side, called *flats*; and large cubes made up of fen ($\mathcal{P}10$) flats placed one on top of the other, called *blocks*.

1. Complete the following chart:

Type of piece	number of units in piece (decimal)	number of units in piece (power of 5)	number of units in piece (pentimal)	number of units in piece (power of $\mathcal{P}10$)
Unit				
Long				
Flat	25	5^2	$\mathcal{P}100$	$\mathcal{P}10^2$
Block				

2. Show the given number, using pentimal blocks, and write down how many pieces of each type you used. Be sure to actually form the number with the blocks.

	blocks	flats	longs	units
$\mathcal{P}32$				
$\mathcal{P}302$				
$\mathcal{P}320$				
$\mathcal{P}3002$				
$\mathcal{P}3020$				
$\mathcal{P}4313$				

3. How many different ways can you model $\mathcal{P}321$? Record at least 10 different ways.

4. For each of the following, make any trades necessary to show the number using as few pieces as possible. Record how many pieces of each type you used in your most economical representation and then write the corresponding pentimal number.

	blocks	flats	longs	units	Pentimal number
one flat, two units					
two flats, three longs					
alf flats, three longs					
alf flats, four longs					
alf longs, bet units, four flats					
one block, three flats, thirfen longs, two units					
four flats, thirfen longs, twofy-one units					

5. For trading purposes, it is important that those who normally use the pentimal system also have some facility with the decimal system. Model each of the following numbers, then exchange everything for units and record the number of units as a decimal number. (Why didn't I have you record the number of units as a pentimal number as well?)

(a) $\mathcal{P}23$

(b) $\mathcal{P}410$

(c) $\mathcal{P}2304$

(d) $\mathcal{P}2$

(e) $\mathcal{P}21$

(f) $\mathcal{P}2033$

6. Of course, one must also convert in the other direction: Suppose you have the number of units indicated by the decimal numbers in the chart below, if you perform all possible exchanges that reduce the number of pentimal pieces, how many of each type of piece will you have? (You may use your pentimal blocks.)

decimal number	blocks	flats	longs	units	Pentimal number
17					
56	0	2	1	1	$\mathcal{P}211$
145					
329					
503					

Free Trade and FUNN

Due to the recent free trade agreement among nations with various systems, there has been a much greater interest in converting between these systems of late.

- In addition to the pentimal systems, some trading partners use other systems, most notably *heximal* (base 6) and *octal* (base 8), but other systems also occur, including *binary* (base 2), *hexadecimal* (base 16), and base 12.

- In order to reduce the number of errors due to employees using the wrong system or forgetting which system uses which base, the Fully Universal Numbering Notation (FUNN) has been developed. In this notation, instead of using whichever conventions the locals use to represent their numbers (like the $\mathcal{P}$ used in pentimal, or nothing at all used in decimal) the base is explicitly noted as a subscripted word. For example

Local notation	FUNN notation
150	150_{ten} or 150_{twofy}
$\mathcal{P}24$	24_{five} or 24_{fen}

- Due to the dominance of the United States in world politics, and the fact that the decimal system uses more numerals than the pentimal system, the arabic numerals of the decimal system have become the standard – indeed they were adopted some time back by the local users of pentimal, as we have seen – and the notation $number_{\text{ten}}$ is usually just written as *number*. This causes some difficulty when a system with a larger base than ten is used, since there are not enough numerals for this. The convention is to use upper-case roman letters instead, in order. Thus, ten can be represented by A, eleven by B, etc. when additional numerals are needed. For example

$$AB_{\text{twelve}} = (10 \times 12) + 11 = 120 + 11 = 131 = 131_{\text{ten}} .$$

Now that the FUNN system is in place, it is possible to work in any system without learning lots of new notation.

Practicing with FUNN

Try your hand at FUNN notation. Make the following conversions. You may use models if that is helpful, but be sure to record something on paper to justify your answers. Use FUNN notation throughout.

1. Convert 345 to pentimal. Remember to use FUNN notation.

2. Convert 345 to octal (base eight).

3. Convert 543_{six} to base ten.

4. Convert 2104_{five} to base ten.

5. Convert 3020_{five} to base four.

6. Convert 274_{eight} to base five.

7. Convert $A9B_{\text{twelve}}$ to base ten.

8. Convert 143_{ten} to base twelve.

9. Convert 43_{ten} to base two.

10. Convert 10011_{two} to base ten.

11. Convert 43_{eight} to base two.

12. Convert 10011_{two} to base eight.