

Number Systems

We have consider several number systems so far this semester. Each has a name and a traditional letter associated with it and often these are set in a special font to indicate that they refer to number systems: \mathbb{N} for the counting numbers,⁴ \mathbb{W} for the whole numbers (the counting numbers plus zero), \mathbb{Z} for the integers,⁵ and \mathbb{Q} for the rational numbers.⁶ In addition, although this is not a commonly studied number system, let's use \mathbb{F} to denote fractions.

1. What is the difference between \mathbb{F} and \mathbb{Q} ? (There is no explanation of this above; you will need to use what you know to come up with a reasonable distinction.) Why is \mathbb{Q} more important than \mathbb{F} ? (You may want to return to this after working on the rest of this sheet.)
2. CLOSURE. One important type of property of a number system is called a *closure* property. A number system is *closed under an operation* if any time you apply the operation to numbers in the system, you get another number in the same system. For each of the four basic arithmetic operations (addition, subtraction, multiplication and division) decide which of the four systems above are closed under that operation.
Can you think of any other operations on numbers? Which systems are closed under you additional operations?
3. FINDING A GO-BETWEEN. For each pair of rational numbers, if possible, find another rational which is between them. (Express your answer as an inequality like $\frac{1}{2} < 0.75 < \frac{7}{8}$.)

a) $\frac{7}{9}; \frac{8}{9}$

b) $\frac{5}{3}; \frac{17}{10}$

c) $\frac{8}{9}; \frac{7}{8}$

d) $\frac{3}{8}; 0.375$

e) $0.3; 0.4$

f) $0.33; 0.32$

g) $\frac{1}{2}; 0.499$

h) $0.333; \frac{1}{3}$

4. DENSITY. If between any two numbers in a number system there is another number from the same system, we say that the number system is *dense*. Which of the number systems above are dense? (If the system is dense, say how you know. If it is not dense, give an example of two numbers with no number between them.)
5. Can every fraction be written as a decimal?
6. Can every decimal be written as a fraction?
7. All decimals on this page that are between 0 and 1 are expressed using a leading zero before the decimal point. Is this necessary? Is it advantageous?

⁴ \mathbb{N} for 'natural'. Note: the counting numbers are really not that important once zero has been encountered, so sometimes \mathbb{N} and 'natural numbers' are used to refer to the whole numbers rather than the counting numbers.

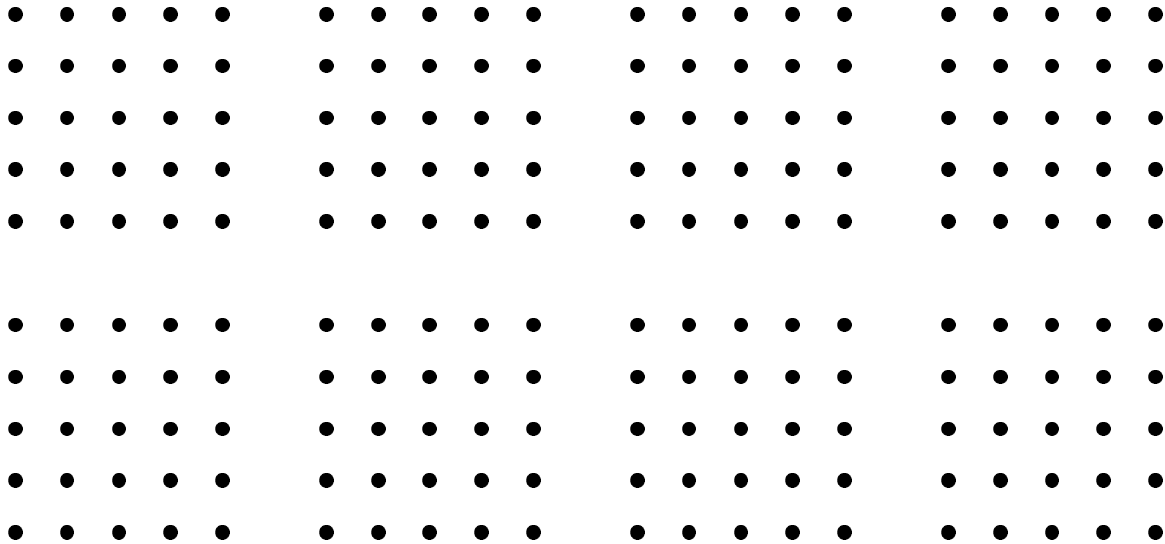
⁵ \mathbb{Z} for the German word *Zahl*.

⁶Why do you think \mathbb{Q} is used?

Irrational Numbers on the Geoboard

A geoboard consists of a flat piece of wood with a grid of nail sticking out of it. Line segments can be represented by looping rubber bands over the nails. The vertical or horizontal distance between two adjacent nails is one unit.

- There are eight different (not the same size) squares that can be formed on a 5 by 5 geoboard. Sketch them. Determine the length of the side and the area for each one. (You may need to use the Pythagorean Theorem to determine the side length.)

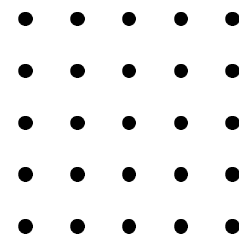
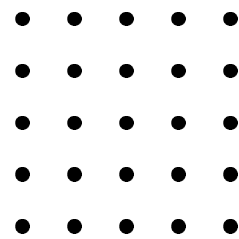
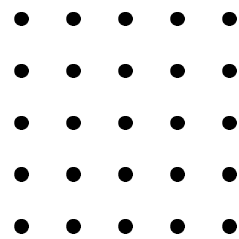


- For each of the pairs of lengths below, (i) form a line segment on your geoboard for each one and record it on the grid, and (ii) use this to determine which is larger (record your answer using $<$, $>$, or $=$).

a) $2\sqrt{2}$ $\sqrt{5}$

b) $3\sqrt{2}$ $2\sqrt{5}$

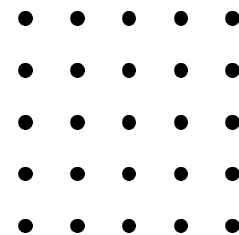
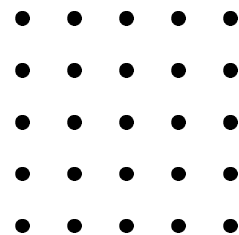
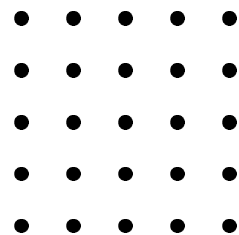
c) $\sqrt{2} + \sqrt{8}$ $\sqrt{10}$



d) $3\sqrt{2}$ $\sqrt{10}$

e) $\sqrt{10}$ $2\sqrt{5}$

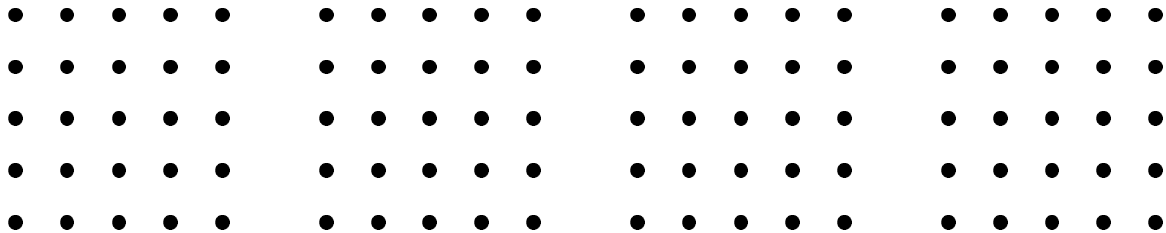
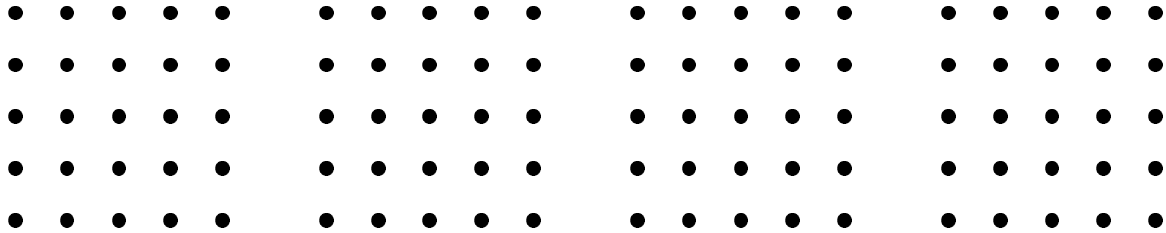
f) $2\sqrt{2}$ $\sqrt{8}$



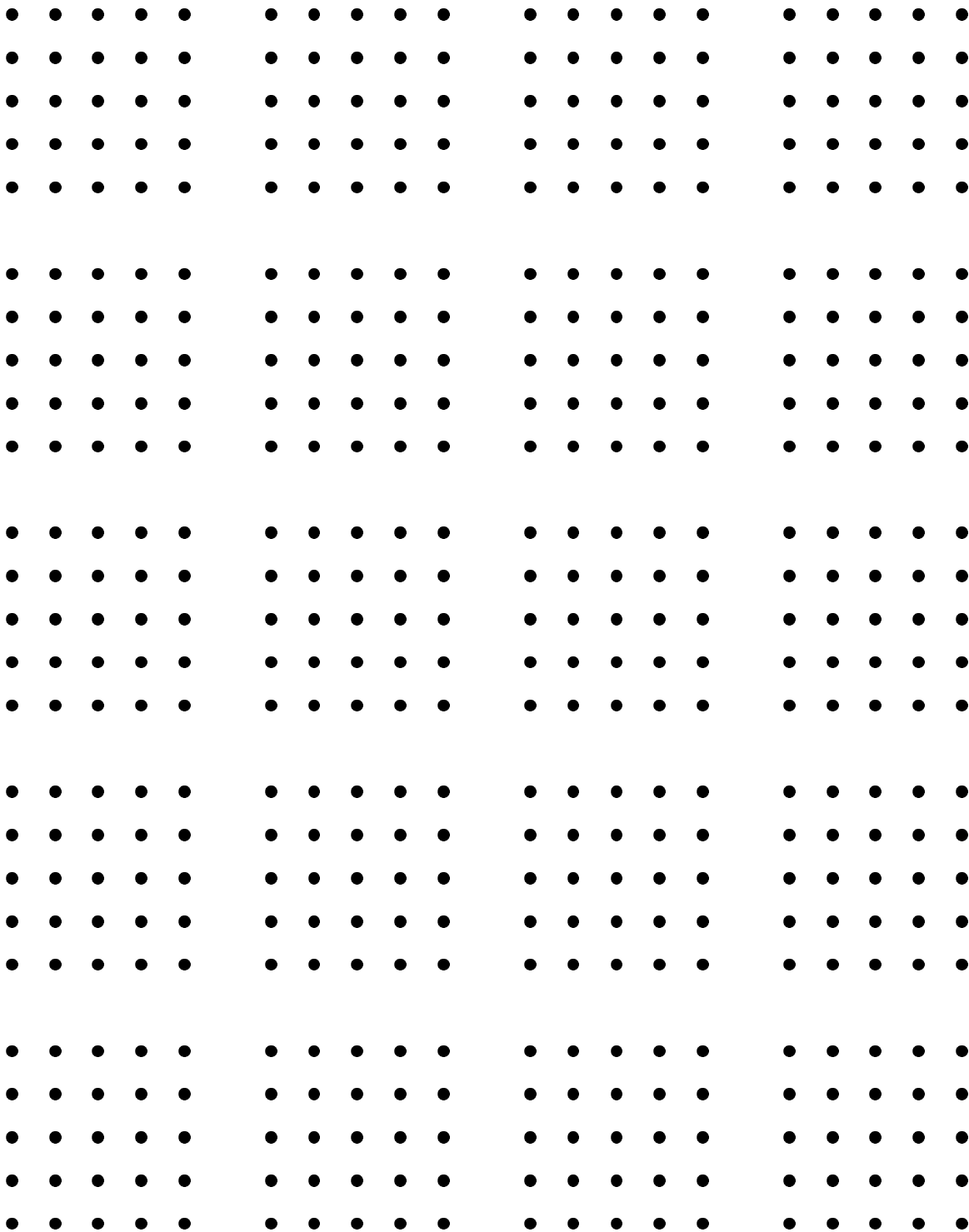
g) Use your answers above to determine which one of the following equations is true for any positive numbers a and b . For the false ones, give examples from above that show they are false.

$$\sqrt{a} + \sqrt{b} = \sqrt{a+b} \quad \sqrt{a \times b} = a \times \sqrt{b} \quad \sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

3. For each of the following polygons, determine the perimeter (the distance around the polygon). Simplify your answer if possible



4. Express each number from 1 to 10 using a number sentence that involves the number 4 exactly four times (and no other numbers). Use at least one square root for each. For example, $48 = (4 + 4) \times (4 + \sqrt{4})$.



Using Algebraic Sentences

1. Write an equation that can be used to calculate the number of feet in a measurement given the number of yards. Use F for the number of feet and Y for the number of yards.
2. I went to the store and bought an equal number of books and CDs. The books cost \$2 each and the CDs cost \$6 each. I spent a total of \$40. What, if anything, is wrong with the following reasoning?

$$2B + 6R = 40$$

$$B = R$$

$$2B + 6B = 40$$

$$8B = 40$$

The last equation says that 8 books is \$40, so each book must cost \$5.

3. Translate the following English sentences into algebraic sentences. Be sure to *carefully* identify each variable used.
 - a) The combined weekly incomes of Fred and Harry total \$490.
 - b) Sally works two jobs on campus. One job pays \$5 an hour, the other pays \$6 an hour. Last week she made \$253 (before taxes).
 - c) Larry is four times as old as his son Bobby.
 - d) If I weighed 10 pounds less than I currently do, I would weigh 150 pounds.
 - e) Four more than five times some number is one less than six times that same number.
 - f) At Mindy's restaurant, for every four people who ordered cheesecake, there were five who ordered pie.
 - g) My age in ten years will be twice my current age. (Use only one variable.)
4. Now use algebra to solve the following problems based on the situations above.
 - a) If Fred makes \$200 in a week, how much does Harry make in a week?
 - b) If Sally worked an equal number of hours at each job, how many hours did she work?
 - c) Five years ago Larry was seven times as old as his son. How old are they now?
 - d) How much do I weigh?
 - e) What is the number?
 - f) If 100 people ordered cheesecake, how many ordered pie?
 - g) How old am I?