Classification of Addition and Subtraction Word Problems

1. Van de Walle describes four types of addition/subtraction problems. What are they? Each of these main types has two or three subtypes. What distinguishes the subtypes?

For problems each problem of 2 – 12 below, determine its type and subtype, and write one or more number sentences associated with the problem, putting a box around the unknown in your number sentence. Your number sentences should include the sentence you feel is most "natural" to the problem as well as a number sentence in *computational form.* Indicate which number sentences are in computational form.

2. Al is 49 inches tall. Bea is 52 inches tall. How much taller is Bea than Al?

3. Deb had 7 cookies. She gave some to Cy. Now Deb has three cookies. How many did she give to Cy?

4. Ed has eight cousins. Three of them are boys. How many girl-cousins does Ed have?

5. After Flo lost five of her new back-to-school pencils, she had 7 pencils left. How many did Flo start with?

6. Hal had two sweaters. Grandma knitted some more for him. Now Hal has 8 sweaters. How many did he get from Grandma?

7. Ida has three more brothers than Jose. Ida has five brothers. How many does Jose have?

8. Kendra had 6 hats. Lea gave her 3 more. How many hats does Ken have now?

9. Maria has 5 trucks and 8 cars in her sandbox. How many vehicles does she have in all?

10. Nadia had twelve wine glasses, but Oliver broke four of them. How many wine glasses does Nadia have left?

11. Pa has five fewer teeth than Ma has. Pa has 8 teeth. How many does Ma have?

12. Rolando had done some of his homework problems, but was stuck on some others. His friend Syd helped him on four more problems. Now Rolando has 11 problems finished. How many did he have finished before getting help from Syd?

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13. Did all of Van de Walle's subtypes occur above? If not, write your own problems for any subtypes that did not occur. Identify the type and subtype of each, and write number sentences for you problems just as you did for problems 2 – 12.

14. Which of the subtypes did you find most natural to express in their computational form?

Meaning of Multiplication and Division

Van de Walle identifies four categories of multiplication/division problems, among which two are especially important: equal groups and multiplicative comparison. Each of these categories has three subcategories, based on what piece of information is unknown.

1. If there are two categories and each has three subcategories, how many classifications by category and subcategory are there? Explain how you arrived at your answer.

2. What type of problem is the problem above?

3. For each type of problem (category and subcategory):

a) write a brief story problem of that type, making sure that no number occurs more than once in the problem or its solution,

b) solve the problem using an array model and one other model (make a sketch of each model on your paper)

c) write down a number sentence for your problem and put a box around the unknown information,

d) write down any related number sentences (again with a box around the missing information).

4. Which of the problems above were hardest to invent?

5. Which of the problems above were hardest to model?

Basic Facts

How do you do your basic facts? For each basic fact below, identify which strategies listed in Van de Walle you use. If different people in your group use different strategies, record all strategies used. If no one uses any of the strategies from Van de Walle, describe some other way of arriving at the basic fact in the space below the table.

Here is a list of strategies for each operation on this sheet and a shorthand notation for each.

Addition: 1-2: 1 & 2 more than; 0: Zero facts; D: doubles; ND: Near Doubles; E10: Elevator (make 10)

Subtraction: 1-2: 1 & 2 less than; D: Doubles/Near Doubles; 0: Zero facts; 10F: Ten frame facts; S10: Subtract 10 and adjust; B10: Bridge through 10

Multiplication: D: Doubles; Z: Zero and One facts; 5: 5 facts; 9: Nifty Nines; H: Helping facts (list the helping fact)

Fact	Strategy	Fact	Strategy	Fact	Strategy
$9+8$		$6-1$		9×6	
$7+5$		$7-2$		6×6	
$6+6$		$4-0$		0×7	
$4+6$		$9-5$		5×3	
$7+6$		$10-7$		8×2	
$8+1$		$8-3$		7×5	
$8+4$		$15-6$		6×8	
$2+4$		$13-8$		4×1	
$5+9$		$9-6$		2×4	
$3+0$		$9-5$		3×7	

Addition Algorithms

1. Solve the following problems using models any way you like. (You might like to see if you can use different methods on each problem.) Have one person manipulate the models and another person record what is going on. Take turns in each role.

a) There were 46 first-graders from Adams Elementary School at the picnic. Then 37 first-graders from Lincoln School arrived. How many first-graders were at the picnic all together?

b) There are 355 boys and 187 girls in the youth soccer program. How many children are participating in the youth soccer program?

c) Jo's bank account had \$1328 in it. Then Jo deposited a check for \$576. How much was in the account after the deposit?

2. Do the following sums using "standard" algorithm.[1] Manipulate the model and record your work as you go, as before, each person taking a turns as manipulator and as recorder.

$$\begin{array}{r} 55 \\ +\ 48 \\ \hline \end{array} \qquad \begin{array}{r} 248 \\ +\ 165 \\ \hline \end{array} \qquad \begin{array}{r} 706 \\ +\ 395 \\ \hline \end{array}$$

3. Do the following additions first without using models, recording each step as it occurs. Then read your steps and simulate the work with a model.

$$\begin{array}{r} 875 \\ +\ 204 \\ \hline \end{array} \qquad \begin{array}{r} 234 \\ +\ 567 \\ \hline \end{array}$$

4. Do the following base five sums. Use the models as you see fit. Is there more than one way to do these?

$$\begin{array}{r} 23_{\text{five}} \\ +\ 34_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 104_{\text{five}} \\ +\ 401_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 1432_{\text{five}} \\ +\ 23_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 1234_{\text{five}} \\ +\ 1432_{\text{five}} \\ \hline \end{array}$$

5. Do the following sums in other bases.

$$\begin{array}{r} 23_{\text{six}} \\ +\ 34_{\text{six}} \\ \hline \end{array} \qquad \begin{array}{r} 103_{\text{four}} \\ +\ 321_{\text{four}} \\ \hline \end{array} \qquad \begin{array}{r} 1234_{\text{nine}} \\ +\ 5678_{\text{nine}} \\ \hline \end{array} \qquad \begin{array}{r} 101101_{\text{two}} \\ +\ 100111_{\text{two}} \\ \hline \end{array}$$

[1] By standard algorithm, I mean the one you were probably all taught in school (starting with the "ones" and moving "right to left"). But note that "standards" have changed over time.

Subtraction Algorithms

1. EXPLORATION. Solve the following problems using models any way you like. (You might like to see if you can use different methods on each problem.) Have one person manipulate the models and another person record what is going on. Take turns in each role.

a) Last week, Jo earned \$423, but \$176 was taken out for taxes, health insurance, etc. After this withholding, how much was Jo's takehome pay?

b) Craig said, "I have 53 keychains in my collection." Diane said, "Oh, I have only 37 keychains." How many more keychains than Diane does Craig have?

By the way, how would each of the problems above be categorized by Van de Walle?

2. STANDARD ALGORITHM. Do the following differences using the models but following the "standard" algorithm (i.e., working from little to big). Manipulate the model and record your work as you go.

56	82	127	430	403
− 34	− 57	− 59	− 234	− 246

3. BASE FIVE. Do the following base five differences. Use the models as you see fit. Is there more than one way to do these?

32_{five}	41_{five}	1421_{five}	4321_{five}
$-\ 24_{\text{five}}$	$-\ 14_{\text{five}}$	$-\ 123_{\text{five}}$	$-\ 1234_{\text{five}}$

4. Do the following sums in other bases.

32_{six}	101_{four}	507_{nine}	101101_{two}
$-\ 14_{\text{six}}$	$-\ 22_{\text{four}}$	$-\ 463_{\text{nine}}$	$-\ 100111_{\text{two}}$

5. Can you figure out what is going on in each of the algorithms below? Try to explain both *what* is happening (procedural description) and *why* it gives the correct result.

243	243	243
− 68	− 68	− 68

Multiplication Algorithms

1. Use base 5 pieces to model the following multiplication problems. For each sketch the model and indicate your regrouping.

$$\begin{array}{r} 22_{\text{five}} \\ \times \quad 2_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 12_{\text{five}} \\ \times \quad 3_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 12_{\text{five}} \\ \times \quad 13_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 12_{\text{five}} \\ \times \quad 23_{\text{five}} \\ \hline \end{array}$$

2. Now try to do the following multiplications without the pieces. Try various algorithms, including the "standard algorithm," the "intermediate algorithm" (what I also call "all pairs"; this has two flavors, depending on whether or not you record the zeros), and the "lattice algorithm". Do each one at least 2 of the three ways. How do your two solutions compare?

$$\begin{array}{r} 32_{\text{five}} \\ \times \quad 3_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 23_{\text{five}} \\ \times \quad 31_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 432_{\text{five}} \\ \times \quad 23_{\text{five}} \\ \hline \end{array}$$

3. Here are some multiplication problems in other bases for you to try. Use any algorithm you like.

$$\begin{array}{r} 22_{\text{four}} \\ \times \quad 3_{\text{four}} \\ \hline \end{array} \qquad \begin{array}{r} 23_{\text{four}} \\ \times \quad 32_{\text{four}} \\ \hline \end{array} \qquad \begin{array}{r} 23_{\text{eight}} \\ \times \quad 16_{\text{eight}} \\ \hline \end{array} \qquad \begin{array}{r} 110_{\text{two}} \\ \times \quad 11_{\text{two}} \\ \hline \end{array}$$

Division Algorithms

1. Use base 5 pieces to model the following division problems.

$22_{\text{five}} \div 4_{\text{five}}$ $121_{\text{five}} \div 4_{\text{five}}$ $344_{\text{five}} \div 3_{\text{five}}$ $1113_{\text{five}} \div 3_{\text{five}}$

2. Now try to do the following divisions without the pieces. (But you may certainly think about what you would be doing with the pieces. You may even want to use the pieces to check your work.)

$3_{\text{five}})\overline{234_{\text{five}}}$ $2_{\text{five}})\overline{1432_{\text{five}}}$ $4_{\text{five}})\overline{1403_{\text{five}}}$ $12_{\text{five}})\overline{1410_{\text{five}}}$

3. Here are some division problems in other bases for you to try.

$2_{\text{four}})\overline{122_{\text{four}}}$ $3_{\text{eight}})\overline{675_{\text{eight}}}$ $4_{\text{six}})\overline{1403_{\text{six}}}$ $12_{\text{six}})\overline{1410_{\text{six}}}$

Algorithm Review

1. BASIC FACTS TABLES. One of the difficulties in doing pencil-and-paper algorithms in other bases is our lack of knowledge of the basic facts expressed in those bases. But we could make a table of these facts and see if that helps us. Fill in the tables below with basic facts for **base 6**.

$+_{six}$	0	1	2	3	4	5
0						
1						
2						
3						
4					12	
5						

$\times_{six}$	0	1	2	3	4	5
0						
1						
2						
3						23
4						
5						

How large would these tables be in base eight? Why?

2. ADDITION. Use the addition facts table and the standard algorithm to compute the following sums. (You should not need to do *any* reasoning in base ten.)

$$\begin{array}{r} 243_{six} \\ +\ 121_{six} \\ \hline \end{array} \qquad \begin{array}{r} 344_{six} \\ +\ 214_{six} \\ \hline \end{array} \qquad \begin{array}{r} 1234_{six} \\ +\ 333_{six} \\ \hline \end{array}$$

3. SUBTRACTION.

a) Explain how to use the addition fact table to find $13_{six} - 4_{six}$.

b) Do the following subtractions twice each, using two different algorithms. [(Algorithms you should know: standard, "9's" complement,[2] elevator (add the same to each number), and the left-to-right algorithm we looked at.] You may want to consult the addition fact table as you do these.

$$\begin{array}{r} 43_{six} \\ -\ 14_{six} \\ \hline \end{array} \qquad \begin{array}{r} 111_{six} \\ -\ 24_{six} \\ \hline \end{array} \qquad \begin{array}{r} 1432_{six} \\ -\ 333_{six} \\ \hline \end{array}$$

[2]Note that in other bases, '9' must be changed to one less than the base. Why?

4. MULTIPLICATION. Multiply using the basic facts tables and the indicated algorithm.

a) Use the "all-pairs" algorithm to compute $21_{\text{six}} \times 34_{\text{six}}$.

b) Use the lattice algorithm to compute $23_{\text{six}} \times 314_{\text{six}}$.

c) Use the standard algorithm to compute $324_{\text{six}} \times 23_{\text{six}}$.

5. DIVISION.

a) Explain how to use the multiplication fact table to determine $23_{\text{six}} \div 3_{\text{six}}$.

b) Use any method you like to do the following division problems.

$$3_{\text{six}} \,)\overline{234_{\text{six}}} \qquad\qquad 4_{\text{six}} \,)\overline{1322_{\text{six}}}$$

6. You can make up additional problems for your self with any base, any operation and any algorithm.