

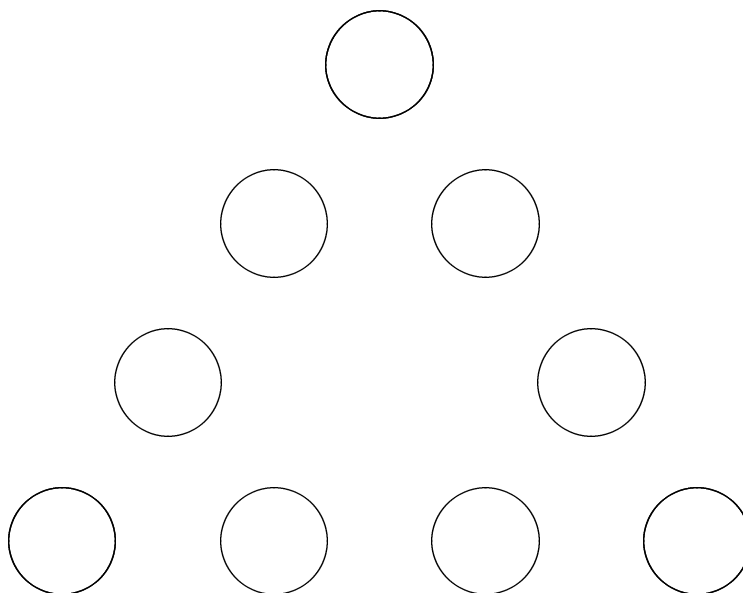
Fill-In the Digits

1. 3-DIGIT SUMS

Using each of the digits 1 through 9 exactly once, fill in the boxes below so that the sum is correct:

$$\begin{array}{r}
 \square \square \square \\
 + \square \square \square \\
 \hline
 \square \square \square
 \end{array}$$

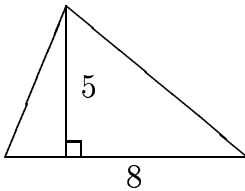
2. 1-9 TRIANGLE PUZZLE



- Using each of the digits 1 through 9 exactly once, fill in the circles of a triangular arrangement similar to the one above so that the sums of the four digits along each side is the same.
- Find as many different solutions as you can in which the side sum is 20. (How should you define “different”? Are some solutions more different than others?)
- Can you find a solution in which the sum on each side is 15? 18? 21? 22?

Procedures and Concepts

1. What is $\frac{3}{4} \div \frac{1}{2}$? Show how to compute it.
2. Write a story problem for which $\frac{3}{4} \div \frac{1}{2}$ would be the appropriate computation to find the solution.
3. Find the area of the triangle below:



4. Why does your method work in problem ???
5. Find $435 \div 18$ without using a calculator.
6. In the following partial computation, why do we “bring down the 5”?

$$\begin{array}{r} 2 \\ 18 \overline{) 435} \\ \underline{36} \\ 75 \end{array}$$
7. In school you were probably taught that “division by zero is not allowed”. Why not?

Some Problems to Solve

Work on the problems on this sheet in two phases:

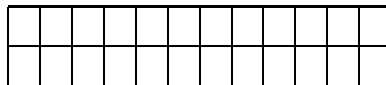
1. First read through all the problems and make a list of ways you might attempt to solve the problem. (Keep in mind the heuristics we have seen.) Jot down some notes for each problem, then move to the next one.
2. Then go back and (attempt to) solve the problems. For each of the problems, explain the reasoning you use to solve the problem. Whether you solve the problem completely or not, keep a list of things you tried, patterns you observed, conjectures you made, facts you discovered, etc. Be sure to note any problem solving strategies you used that did not occur to you when you first read the problem (before working on it).

You may work the problems in any order.

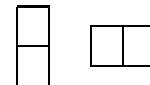
1. What is the last digit of 3^{99} ?
2. How many rectangles (of any size) are there in the picture below?



3. Jan is having a party, to which she invited 24 people. She plans to serve the meal on card tables, arranged in a long rectangle, with each table pushed up against the next. If the card tables are only large enough to seat one person on a side, how many tables will Jan need?
4. How many ways are there to cover the large rectangle below using “dominoes”. (A dominoe is a rectangle that is 1 square wide and 2 squares tall.)



Dominoes:



[Hints: That large rectangle is pretty large, perhaps you should try a some smaller examples first. How can you use smaller examples to help you with the original problem?]

5. A pail with 40 marbles in it weighs 175 grams. The same pail with 20 marbles in it weighs 95 grams. How much does the pale weigh alone? How much does one marble weigh alone?
6. How many ways are there to make change for 27 cents using (any number of) pennies, nickels, dimes and quarters.

Some More Problems

1. What is the last digit of 13^{99} ?
2. A regular pentagon is a figure with five equal (straight) sides and five equal angles. How many diagonals are there in a regular pentagon? (The diagonals are the line segments that connect one corner with another but are not sides. All together they look like a star.)
Can you generalize this for other polygons?
3. How many ways are there to make a path connecting adjacent letters in the diagram below so that the path is labeled with the alphabet (each letter exactly once and in order)?

A
 ABA
 ABCBA
 ABCDCBA
 ABCDEDCBA
 ABCDEFEDCBA
 ABCDEFGFEDCBA
 ABCDEFGHGFEDCBA
 ABCDEFGHIHGFEDCBA
 ABCDEFGHIJIHGFEDCBA
 ABCDEFGHIJKJIHGFEDCBA
 ABCDEFGHIJKLKIHGFEDCBA
 ABCDEFGHIJKLMLKJIHGFEDCBA
 ABCDEFGHIJKLMNMLKJIHGFEDCBA
 ABCDEFGHIJKLMNONMLKJIHGFEDCBA
 ABCDEFGHIJKLMNQPONMLKJIHGFEDCBA
 ABCDEFGHIJKLMNQPQRQPONMLKJIHGFEDCBA
 ABCDEFGHIJKLMNQPQRSRQPONMLKJIHGFEDCBA
 ABCDEFGHIJKLMNQPQRSTSRQPONMLKJIHGFEDCBA
 ABCDEFGHIJKLMNQPQRSTUTSRQPONMLKJIHGFEDCBA
 ABCDEFGHIJKLMNQPQRSTUVUTSRQPONMLKJIHGFEDCBA
 ABCDEFGHIJKLMNQPQRSTUVWVUTSRQPONMLKJIHGFEDCBA
 ABCDEFGHIJKLMNQPQRSTUVWXWVUTSRQPONMLKJIHGFEDCBA
 ABCDEFGHIJKLMNQPQRSTUVWXYXWVUTSRQPONMLKJIHGFEDCBA
 ABCDEFGHIJKLMNQPQRSTUVWXYZZYXWVUTSRQPONMLKJIHGFEDCBA

4. There are 20 people in a room. If each person shakes hands with each other person exactly once, how many handshakes occur?
5. How many squares (of any size) are there on an 8×8 checkerboard?

Find the Pattern

1. For each of the following patterns,

- fill in the blanks,
- describe the pattern using words,
- decide if it is a repeating pattern or a growing pattern,
- see if you can determine the 10th and 50th numbers in the pattern,
- see if you can express the n th number in the pattern. (For some this may be *very* hard, for others it is much easier.)

(a) 2, 4, 6, 8, _____, _____, ...

(b) _____, 5, 8, 11, 14, _____, _____, ...

(c) 5, 10, 20, 40, 80, _____, _____, ...

(d) 3, 10, 7, 3, 4, 1, _____, _____, _____, ...

(e) 2, 5, 7, 12, 19, _____, _____, ...

2. SOME SUMS. Consider each of the following patterned sums.

- a) $1 + 2 + 3 + \cdots + n$. (adding all numbers from 1 to n)
- b) $a + (a + 1) + (a + 2) + \cdots + b$. (adding all numbers from a to b)
- c) $1 + 3 + 5 + \cdots + n$. (adding consecutive odd numbers)
- d) $2 + 4 + 6 + \cdots + n$. (adding consecutive even numbers)

For each of these sums do the following:

- Determine the sum when n is 1, 2, 3, 4 and 5. (Example: for ?? the first 5 terms are 1, 3, 6, 10, 15. For ?? you will need to choose values for a and b .)
- See if you can determine the value of the sums when $n = 10$ and when $n = 50$, or in the case of ?? when $a = 10$ and $b = 50$.
- See if you can express the value of the n th sum using a general formula for n .

How sure are you of your formula(s)? What is the basis of your (level of) belief in your formulas? What would make you more confident?

Do you see any relationships between the various sums? Do the *results* of one part help you with any others? Do the *methods* of one part help you with any others?

3. DIAGONALS. How many diagonals are there in a regular pentagon? (A regular pentagon is a figure with five equal (straight) sides and five equal angles.) Can you generalize this for other polygons?

A-Blocks

1. WHAT DO YOU HAVE? Sort the A-blocks in some convenient way. What attributes and values for these attributes enable these blocks to be distinguished?

2. NAME THE BLOCKS. Devise a convenient naming system for the blocks so that you can record your work in the subsequent activities. For example, you might use R to abbreviate red and T to abbreviate triangle.

3. DIFFERENCE LOOPS. (This is like Activity 18.1 in VdW.) Construct a loop of 10 blocks, each differing from each adjacent piece on exactly one attribute. Record the loop here.

