

## Math 381/CS 360: Test 2 Info

Test 2 will be on Tuesday, April 17. It is by necessity cumulative, but will focus on material since shortly before Test 1. You can consult the course calendar for details about daily topics and book sections, and you can consult old homework for example problems.

In addition to these things here are a few (mostly redundant) comments/reminders:

1. Don't forget about Computability. This was discussed in class just before Test 1 but was not tested on that test. You should know
  - How to “trace” the running of a Turing machine
  - How to make use of Church's thesis in proofs about computability
  - The definitions of computable and computably enumerable, how to show various sets are one or the other, and how to show results relating the two definitions (see PS 10, for example)
2. FOL, quantifiers and all
  - (a) Major Theorems/Results:
    - Soundness (including proof for full FOL)
    - Completeness (statement only for full FOL)
    - Compactness (including proof from Soundness and Completeness, applications)
  - (b) Proof rules, syntax and semantics
  - (c) Tautology, first order consequence, etc.
  - (d) Important ideas from model theory:
    - Definitions of model, satisfaction, variable assignment, etc.
    - Idea of extending the language (by adding constants, for example) and then extending a model correspondingly. (This came up in the non-standard model of PA, proof of soundness, Proposition 1, etc.)
3. Set Theory
  - Naive set theory
    - Axioms (extension and comprehension)
    - Why naive set theory is unsatisfactory (Russel Paradox)
  - ZF
    - Intuition behind axioms (need to restrict comprehension somehow; try to avoid sets that are “too big” or “too complicated”, but still have enough sets around to be useful)
    - Useful mathematical objects that can be constructed in ZF (unions, pairs, relations, functions, models, etc.).
    - How Regularity precludes certain “weird” sets.
  - Special sets: empty set, powersets,  $\omega$
4. Peano Arithmetic and Induction
  - Proofs by induction, especially structural induction
  - Why various forms of induction are all equivalent
  - How to prove basic facts about the natural numbers from Peano's axioms (like in PS 13).