Math 362 – Exam 3 Study Sheet

The most important major theorems for which you should know the proofs are marked with ♢. Of course, you may also be asked to proof other smaller results that we have seen or new results that are based on the techniques we have been learning.

Basic Sequence and Series stuff (see Test 2 review sheet)

- especially uniform vs pointwise convergence

Power Series

1. Weierstrass Theorem (7.26, statement, relation to power series)
2. Radius of convergence (3.39)
3. Uniform convergence in interior of interval of convergence
4. Derivatives and continuity (8.1–8.2)
5. ♢ Taylor's Thm (5.15, proof, relationship to Mean Value Thm)
   - using Taylor's Thm to show Taylor series converge to desired function (like e^x, $\sin(x)$, $\cos(x)$)
   - ♢ proof that e is irrational (3.32)
6. Products of series
   - Definition (3.49)
   - Sufficient condition to imply convergence (3.50)
   - ♢ Using 8.2 to prove 3.51 (see page 175)

Lebesgue Measure and Integration

1. Desired properties for Lebesgue measure and integration
2. ring, σ-ring of sets (11.1)
3. set functions: additive, countably additive, regular (11.2, 11.3, 11.5)
4. construction of outer measure, measurable sets (11.7, 11.9)
5. properties of outer measure, measurable sets (11.8, 11.10, 11.11)
   - Borel sets
   - sets of measure zero
6. measure space, measurable space (11.12)
7. measurable functions and their properties (11.13–11.18)
8. definition of Lebesgue integral, $\mathcal{L}(\mu)$ (11.19, 11.21, 11.22)
   - intuition behind definition
   - ♢ role of simple functions, approximation theorem (11.20)
   - definition for non-negative measurable functions f
   - how to deal with measurable functions that are not non-negative
9. Basic properties of Lebesgue integral (11.23, 11.24)
10. ♢ Lebesgue's monotone convergence theorem (11.28)
    - ♢ applications: 11.29, 11.30, Fatou's theorem (11.31)
11. ♢ Lebesgue's dominated convergence theorem (11.32)
12. ♢ comparison of Lebesgue integration and Riemann integration (11.33)