Math 362
Spring, 2002

Name ________________________________

Test 1 – In Class Portion

1. a) Define what it means for a set E to be ***open*** in a metric space X.

b) Define what it means for a set E to be ***closed*** in a metric space X.

c) Prove that if E is closed, then $X - E$ is open. (The converse is true, but you need not prove it.)

2. a) Define what it means for a sequence $\{p_n\}$ to ***converge*** in a metric space X.

b) Let $\{a_n\}$ be a bounded, increasing sequence of real numbers. Prove that $\{a_n\}$ converges.

3. a) Define what it means to say that a subset K of a metric space X is ***compact***.

b) Give **three** equivalent definitions of "f is a continuous function from X to Y" (where X and Y are metric spaces). One definition should mention limits, the other two should not.

c) Prove that if $f : X \to Y$ is continuous and K is a compact subset of a metric space X, then $f(K)$ is a compact subset of Y.

4. a) State the Heine-Borel Theorem.

b) Outline the proof of "half" of the Heine-Borel Theorem. (This should take $\leq \frac{1}{2}$ page. Once you have a statement, call me over, and I will tell you which half to do.)

5. Prove or give a counterexample: If $\{a_n\}$ is a sequence such that for all n $a_n < 2$, then $\limsup a_n < 2$.

6. Prove the following: If $|a_n| < c_n$ for all $n \in \mathbb{N}$ and Σc_n converges, then Σa_n converges.

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Test 1 – Take Home Portion

7. Let $\{s_n\}$ be a sequence of real numbers and define $a_n = s_{2n}$ and $b_n = s_{2n+1}$. Suppose that $\{a_n\}$ and $\{b_n\}$ both converge (but not necessarily to the same thing). What can be said about the subsequential limits of $\{s_n\}$? Justify your answer.

8. Let E' denote the set of all limit points of E.

a) Show that for any subsets A and B of a metric space X, $(A \cup B)' = A' \cup B'$.

b) Find an example where $(A \cap B)' \neq A' \cap B'$.

9. Prove the following: If K is a compact subset of a metric space X, then K is bounded. (Note: you cannot apply Heine-Borel, since X might not be Euclidean. This result shows that one direction of the the Heine-Borel Theorem is always true, since we know that K must be closed.)