

## Math 362 – Test 1 Study Sheet

### Metric Spaces

1. definition of a metric space
2. important terms and their definitions (2.18)
3. important examples of metric spaces and sets in them with various properties
4. important theorems
  - 2.23: complement of an open set is closed and vice versa
  - 2.24: unions and intersections of open/closed sets
  - 2.27: closure of a set ( $\overline{E}$ )
  - 2.28: a condition that implies  $\sup E \in \overline{E}$
  - 2.30: relative topology

### Compact Sets

1. definitions of open cover, compact set
2. theorems 2.33, 2.34, 2.35
3. finite intersection property (thm 2.36)
4. nested  $k$ -cells (thms 2.38, 2.39)
5.  $k$ -cells are compact (thm 2.40)
6. Thm 2.41 (Heine-Borel Theorem, how previous theorems are used in proof)

### Sequences

1. definition of convergence of sequence in a metric space
2. subsequences (and  $\liminf$ ,  $\limsup$ )
3. limits and algebra (thms 3.3 and 3.4)
4. Cauchy sequences (definition, thm 3.11, complete metric spaces)
5. diameter of a set, thm 3.10

### Series

1. partial sums, definition of convergence
2. Cauchy criterion for series (thm 3.22)
3.  $p$ -series, geometric series, definition of  $e$
4. Convergence Tests (thms 3.22, 3.23, 3.25, 3.27, 3.33, 3.34)
5. Power series, radius of convergence

### Continuity

1. definition of limits, continuous functions in metric spaces (thm 4.2)
2. characterization of continuous functions in terms of open sets (thm. 4.8)
3. continuous images of compact sets are compact (thm 4.14)

### General

1. important examples
2. methods of proof
3. pictures to accompany theorems (it will help you remember statements and proofs)