February 17 – Randomness

1. Random “experiments.” Essential features:
   (a) more than one possible outcome,
   (b) the experiment is repeatable under (more or less) identical conditions,
   (c) the outcome that will obtain under any given repetition is uncertain.

2. Two canonical examples related to data collection.
   (a) A random sample of size \( m \) is to be chosen from a finite population of size \( n \),
   (b) A number \( n \) of units are to be assigned at random to \( k \) experimental treatments.

3. Crucial terminology. Fix an experiment \( E \).

   **Definition 1.** An **outcome** is one of the possible (atomic) results of the experiment.

   **Definition 2.** The **sample space** is the set of possible outcomes.

   **Definition 3.** An **event** is any subset of the sample space.

4. Example: for the first canonical example. Suppose that a random sample of size 30 is taken from the population of all Calvin senior students. Any specific collection of seniors of size 30 is an outcome. (There are boatloads of these.) Some example events include: the set of samples that have exactly 15 males and 15 females; the set of samples that have exactly 10 students with GPA greater than 3.00; the set of samples that have 30 varsity basketball players (there aren’t too many of these!).

5. A **probability function** is a rule that assigns to each event \( A \) of the experiment a real number, \( P(A) \) such that \( 0 \leq P(A) \leq 1 \).

6. The probability \( P(A) \) is supposed to “predict” the long-run relative frequency of the occurrence of the event \( A \) if the experiment is repeated many times. In other words:

   \[
   P(A) \approx \frac{\text{number of times } A \text{ occurs}}{\text{number of times experiment is repeated}}
   \]

   with the approximation improving as the number of times the experiment is repeated increases.

7. Important:

   Probability statements are about what may occur in the long-run and are statements before the fact about what the result of the process of doing the experiment might be. They are not statements about what has already occurred. What has already occurred is certain!

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**Homework, Due Tuesday, February 22**

1. Read Devore and Farnum, Section 5.1.

2. Do problems 5.1,2,3,4,5 of Devore and Farnum.

3. Suppose that two numbers are to be chosen at random (without replacement) from the numbers 1, \ldots, 10.
   (a) Make a systematic list of all the outcomes of this random experiment. How many are there?
   (b) Let \( A \) be the event such that the sum of the two numbers chosen is even. List the outcomes in \( A \).
   (c) For the event \( A \) in part (b), what is a reasonable number to define \( P(A) \) to be? Why?
   (d) Now suppose that the two numbers are to be chosen at random with replacement. How many outcomes are there (you need not list them)? what is a reasonable number to assign \( P(A) \) in this case?