Math 221C
Fall 1999

Name ____________________
Group ______

Test 5

Instructions. Do not work on the test sheets; use the blank paper provided. Each sheet should be clearly labeled with your name and the question(s) being answered. Please use only one side of each sheet of paper. When you are finished, put your sheets in order and bring them up to be stapled together. **Do not write within 1 inch of the upper left corner of each page;** anything you write in that corner gets covered up by the staple.

All of the questions are either short answer or mathematical tasks. In grading **short answer** questions I will generally be looking for answers that are true, accurate, **concise**, coherent, important, and supported. Be sure to show all of your work on the **mathematical tasks**, **explaining your reasoning as you go**. If you use a calculator, be sure to record the **operation and result**.

You do not need to work on the problems in order, but please arrange your work on the paper in such a way that it can be put in order when you turn it in.

1. CONCEPTUAL LEARNING. Van de Walle claims that one of the best ways to ensure that students develop a deep conceptual understanding of mathematics is to force them to translate between various representations for the same thing. (In class we have sometimes referred to this as the "big triangle" or the "big star".) **[6]**

List three of the five types of representation that Van de Walle has in mind.

2. THE KEY TO PERCENTS. What is the key to a conceptual understanding percents? (A three word answer is sufficient.) **[4]**

3. DECIMAL-FRACTION-PERCENT CONVERSIONS. Do the following conversions. All answers should be *exact*. **[12]**

a) 3.4% to a decimal and to a fraction.
b) $\frac{7}{22}$ to a decimal and to a percent.
c) $1.3\overline{2}$ to a fraction and to a percent.

4. DIVISION WITH FRACTIONS. **[13]**

a) Use two different pencil-and-paper algorithms to compute $\frac{9}{4} \div \frac{2}{3}$. (Show enough work that I can tell how the algorithm is working.)
b) Pick one of these two algorithms and explain *why* it works.
c) Write a word problem for which computing $\frac{9}{4} \div \frac{2}{3}$ provides the answer. Be sure to explain how you interpret the numerical answer in relation to your word situation.

5. DROPOUT RATES. The dropout rate at Tough University is 18% during the freshman year, and 5% during each of the remaining three years. What percentage of the incoming class at TU graduates four years later? **[5]**

6. BAGS OF CANDY. Solve the following problem using **two different methods**: **[5]**

> Four bags of candy contain a total of 100 pieces of candy. How many pieces of candy are in 12 bags?

7. INTEGER MODELS. Show how to model $-3+(-4)=-7$ and $-3-4=-7$. Be sure to label which is which. Your diagrams should make it clear exactly how a student would model this with materials. (I don't just want a "snapshot" of what the model looks like when the computation is finished.) **[5]**

8. NEGATIVES AND POSITIVES. Use the meaning of multiplication to explain why "a positive times a negative is a negative." Use the example of 4×-3 to illustrate your explanation. **[5]**

9. BROKEN DECIMAL POINT. You are taking an exam and the decimal point on your "four-function" calculator breaks. (A four-function calculator has only 4 arithmetic operations: addition, subtraction, multiplication and division in addition to the digits and an equals sign.) If you press the decimal point key nothing happens. Furthermore, the decimal point does not display on the screen either. (For example, if you type $10 \div 4$, the calculator answers 25.) Fortunately everything else still works. Explain how you can calculate each of the following values **on a four-function calculator with no decimal point**. Show each operation you would have the broken calculator perform and what the answer would be on the broken calculator. Use notation like in the example below. **[10]**

a) $2.34 + .987$ b) 0.35×4.8 c) $5.43 \div .123$

Example notation: $\boxed{1}\,\boxed{0}\,\boxed{\div}\,\boxed{4}\,\boxed{=}$ results in 25 (Each box should contain one of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, $+$, $-$, $\times$, $\div$, or $=$. No other operations are allowed. Be sure to explain how you get the correct answer from your results.)

HINT: Think about *how* and *why* the standard pencil-and-paper algorithms work.

[10]

10. PENTIMAL NOTATION FOR NUMBERS LESS THAN 1. We did not discuss extending our base five system (also called the pentimal system) to the right of the "pentimal point", but we could have. Use your knowledge of how place value works to answer the following questions.

a) Express the number $.2_{\text{five}}$ using base five place value language.

b) Write the number $.2_{\text{five}}$ as a fraction and as a base 10 decimal. Explain your answer.

c) Write the number 2.31_{five} as a fraction and as a base 10 decimal. Explain your answer.

d) Write $\frac{1}{3}_{\text{ten}}$ as a base five "pentimal".