

Math 221C  
Fall 1999

Name \_\_\_\_\_  
Group \_\_\_\_\_

Test 4

**Instructions.** Answer the questions below on the paper provided. **Do not do any work on the test paper.** Each sheet should be clearly labeled with your name and the question(s) being answered. Please use only one side of each sheet of paper. When you are finished, put your sheets in order and bring them up to be stapled together. **Do not write within 1 inch of the upper left corner of each page;** anything you write in that corner gets covered up by the staple.

All of the questions are either short answer or mathematical tasks. In grading **short answer** questions I will generally be looking for answers that are true, accurate, **concise**, coherent, important, and supported. Be sure to show all of your work on the **mathematical tasks**, **explaining your reasoning as you go.** If you use a calculator, be sure to record the operation and result.

You do not need to work on the problems in order, but please arrange your work on the paper in such a way that it can be put in order when you turn it in.

1. GCD AND LCM. Find the greatest common divisor (GCD) and least common multiple (LCM) of 54 and 180. [8]
2. MODELS FOR FRACTIONS. [10]
  - a) Draw a picture that **clearly** shows that  $\frac{4}{5} = \frac{12}{15}$ .
  - b) Van de Walle distinguishes three types (or categories) of models for fractions, what are the three types?
  - c) Which type of model is your picture for part (a)?
3. FRACTIONS. [10]
  - a) Using language a fifth-grader could understand, explain what the top and bottom numbers of the fraction  $\frac{5}{9}$  mean.
  - b) A child is using a model sketched below to determine  $\frac{2}{3} + \frac{1}{4}$ :
 

The child concludes that  $\frac{2}{3} + \frac{1}{4} = \frac{3}{7}$ . Explain why this reasoning is incorrect. **Do not merely show how to do the problem correctly, but identify the error being made by the student.** Your answer should make it clear that you understand what the child was probably thinking.
  - c) What is the correct value of  $\frac{2}{3} + \frac{1}{4}$ ?
4. AREA MODEL FOR MULTIPLICATION OF FRACTIONS. Sketch an area model for  $\frac{5}{3} \times \frac{4}{7}$  and **indicate how the model can be used to determine the value of the product.** [7]

5. COMPARING FRACTIONS. For **TWO** of the following, explain a **conceptually based thinking strategy** (not an algorithm like “cross-multiply” or “common denominators”, not by using models alone) for determining which fraction is larger. [6]

a)  $\frac{2}{9}$   $\frac{2}{7}$

b)  $\frac{4}{9}$   $\frac{7}{13}$

c)  $\frac{5}{7}$   $\frac{7}{9}$

6. PATTERN BLOCKS. [6]

- a) Use pattern blocks to model  $\frac{5}{3} \div \frac{1}{2}$ . **Use two yellows for the whole.** Write down the colors of the blocks you use to represent things (you may want to sketch them as well) and how you interpret the model numerically.
- b) When you do division with this model, which interpretation of division are you using: fair share, repeated subtraction, or missing factor? Explain.

7. LOCKERS. The two janitors from our locker problem are at it again. Just as before the first janitor opens each of the 100 lockers, then the second janitor changes every second locker (lockers 2, 4, 6, etc.), then the other janitor changes every third locker (lockers 3, 6, 9, etc.), and so on just as in class. But this time, the the supervisor walks in immediately after round 20 (when one of the janitors has just changed the door of lockers 20, 40, 60, 80, and 100). The janitors immediately stop their locker game and get back to work. Which of the following lockers are open and which are closed when they interrupt the game? (Be sure to give some indication of how you know). [8]

- a) locker 16  
b) locker 19  
c) locker 81  
d) locker 92

8. TWO PROBLEMS. [15]

- a) Find **two** numbers that each have exactly 30 factors. Explain how you found the numbers and how you know that they have exactly 30 factors.
- b) One night Alexandra was studying for her exams. She spent  $\frac{2}{5}$  of her time studying for her math final,  $\frac{1}{6}$  of her time studying for her biology final,  $\frac{7}{20}$  of her time studying for her religion final, and the remaining 25 minutes studying for her Spanish final. How much time did she spend studying for finals that night?