

Math 221C
Fall 1999

Name _____
Group _____

Test 3

Instructions. Answer the questions below on the paper provided. **Do not do any work on the test paper.** Each sheet should be clearly labeled with your name and the question(s) being answered. Please use only one side of each sheet of paper. When you are finished, put your sheets in order and bring them up to be stapled together. **Leave a 1 inch margin at both sides of the paper and some space between problems.**

All of the questions are either short answer or mathematical tasks. In grading **short answer** questions I will generally be looking for answers that are true, accurate, **concise**, coherent, important, and supported. Be sure to show all of your work on the **mathematical tasks**, **explaining your reasoning as you go**. If you use a calculator, be sure to record the operation and result.

You do not need to work on the problems in order, but please arrange your work on the paper in such a way that it can be put in order when you turn it in.

1. BASE FIVE ARITHMETIC. Compute the following working in base five using any means you like other than converting everything to base ten and back again. You may use the models or make sketches of them if you like, but **you must also record your work using a recording scheme** (a.k.a. pencil-and-paper algorithm). [12]

a) $312_{\text{five}} + 144_{\text{five}}$

b) $42_{\text{five}} \times 23_{\text{five}}$

c) $412_{\text{five}} \div 3_{\text{five}}$

2. A LITTLE BIT IN BASE SEVEN. [12]

a) Use the “standard algorithm” to compute $432_{\text{seven}} - 345_{\text{seven}}$. **Do not write anything in base 10 as part of your answer.** (If you need to do some reasoning in base 10, write notes off to the side, but show the algorithm as it would be done by someone comfortable with base 7.)

b) Now compute $432_{\text{seven}} - 345_{\text{seven}}$ **using a different algorithm.** Once again, do not write anything in base 10 as part of your answer.

c) Convert 321_{seven} to base 10.

3. STANDARD MULTIPLICATION ALGORITHM. Below is a record of someone using the standard algorithm to compute $26_{\text{ten}} \times 43_{\text{ten}}$ (in base ten). [6]

$$\begin{array}{r} 26 \\ \times 43 \\ \hline 78 \\ 104 \\ \hline 1118 \end{array}$$

- a) What do the 78 and 104 represent?
b) Explain why we add in the last step.

4. OTHER MULTIPLICATION ALGORITHMS. Read all the parts before beginning. [15]
- Compute $26_{\text{ten}} \times 43_{\text{ten}}$ **using the lattice algorithm.**
 - Compute $26_{\text{ten}} \times 43_{\text{ten}}$ **using the all pairs algorithm.**
 - Sketch a picture showing how base ten blocks can be used to model $26_{\text{ten}} \times 43_{\text{ten}}$.
 - Clearly show how this model relates to **both** the all pairs and lattice algorithms.
 - We have now done $26_{\text{ten}} \times 43_{\text{ten}}$ using three different algorithms. **Pick two of them** and compare them, stating the advantages and disadvantages of each when compared with the other.
5. A DIVISION ALGORITHM. A student is performing a division operation (in base ten). So far the student has written: [10]

- What does the 54 indicate? (54 of what?)
- What does the 40 mean? How did the student probably come up with that number?
- What does the 2160 mean and where did it come from?
- What does the 715 mean and where did it come from?
- In your explanations above what interpretation(s) of division did you make use of? (If you used more than one, say where you used each.)
- Finish the computation and record the answer.

6. PROBLEM SOLVING. The following will be graded as problem solving tasks, so clearly show your work, reasoning and ideas, even if you do not obtain a complete solution. Make it clear that you understand the problem, what your plan for solving it is, and how you are progressing with your plan, etc. Feel free to use your calculator to help with the arithmetic, but be sure to record any results you obtain from the calculator and to explain your reasoning. [15]
- Matt Maddox is designing an exercise for his students who are learning to do division using models and recording their work, much as we saw in the video. He wants to talk about remainders, and he wants an answer that is easily remembered so that he can easily check if students (who will be working together in groups) are getting the correct result. He decides he would like the answer to be 123 with a remainder of 4. Come up with a division problem such that
 - the answer is 123 with remainder of 4, and
 - the divisor (the number you “divide by”) is a one-digit number.
 - Describe a general way of making such division problems with any answer you (or Matt Maddox) like.
 - Using the digits 5, 6, 7, 8, and 9 each exactly once, fill in the boxes below to make the largest possible product. Be sure to explain how you come up with your answer and **why you know there are no larger products possible.**

$$\begin{array}{r} \square \square \square \square \\ \times \quad \square \\ \hline \end{array}$$