Irrational Numbers on the Geoboard

A geoboard consists of a flat piece of wood with a grid of nail sticking out of it. Line segments can be represented by looping rubber bands over the nails. The vertical or horizontal distance between two adjacent nails is one unit.

1. There are eight different (not the same size) squares that can be formed on a 5 by 5 geoboard. Sketch them. Determine the length of the side and the area for each one. (You may need to use the Pythagorean Theorem to determine the side length.)

2. For each of the pairs of lengths below, (i) form a line segment on your geoboard for each one and record it on the grid, and (ii) use this to determine which is larger (record your answer using <, >, or =).

   a) $2\sqrt{2}$  \hspace{5pt} $\sqrt{5}$  
   
   b) $3\sqrt{2}$  \hspace{5pt} $2\sqrt{5}$  
   
   c) $\sqrt{2} + \sqrt{8}$  \hspace{5pt} $\sqrt{10}$  
   
   d) $3\sqrt{2}$  \hspace{5pt} $\sqrt{10}$  

   e) $\sqrt{10}$  \hspace{5pt} $2\sqrt{5}$  

   f) $2\sqrt{2}$  \hspace{5pt} $\sqrt{8}$  


g) Use your answers above to determine which one of the following equations is true for any positive numbers \(a\) and \(b\). For the false ones, give examples from above that show they are false.

\[
\sqrt{a} + \sqrt{b} \neq \sqrt{a + b} \quad \sqrt{a \times b} = a \times \sqrt{b} \quad \sqrt{a} \times \sqrt{b} = \sqrt{a \times b}
\]

\[3.\] For each of the following polygons, determine the perimeter (the distance around the polygon). Simplify your answer if possible.

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\[4.\] Express each number from 1 to 10 using a number sentence that involves the number 4 exactly four times (and no other numbers). Use at least one square root for each. For example, \(48 = (4 + 4) \times (4 + \sqrt{4})\).
Using Algebraic Sentences

1. Write an equation that can be used to calculate the number of feet in a measurement given the number of yards. Use \( F \) for the number of feet and \( Y \) for the number of yards.

2. I went to a discount store and bought and equal number of books and CDs. The books cost $2 each and the CDs cost $6 each. I spent a total of $40. What, if anything, is wrong with the following reasoning?

\[
\begin{align*}
2B + 6C &= 40 \\
B &= C \\
2B + 6B &= 40 \\
8B &= 40
\end{align*}
\]

The last equation says that 8 books is $40, so each book must cost $5.

3. Translate the following English sentences into algebraic sentences. Be sure to carefully identify each variable used.
   a) The combined weekly incomes of Fred and Harry total $490.
   b) Sally works two jobs on campus. One job pays $5 an hour, the other pays $6 an hour. Last week she made $253 (before taxes).
   c) Larry is four times as old as his son Bobby.
   d) If Fred weighed 10 pounds less than he currently does, he would weight 150 pounds.
   e) Four more than five times some number is one less than six times that same number.
   f) At Mindy’s restaurant, for every four people who ordered cheesecake, there were five who ordered pie.
   g) My age in ten years will be twice my current age. (Use only one variable.)

4. Now use algebra to solve the following problems based on the situations above.
   a) If Fred makes $200 in a week, how much does Harry make in a week?
   b) If Sally worked an equal number of hours at each job, how many hours did she work?
   c) Five years ago Larry was seven times as old as his son. How old are they now?
   d) How much does Fred weigh?
   e) What is the number?
   f) If 100 people ordered cheesecake, how many ordered pie?
   g) How old am I?
Solutions

Classification of Addition and Subtraction Word Problems

1. Join, Separate, Part-Part-Whole, Compare. Subtypes depend on what piece of information is unknown.

2. Compare: Difference Unknown. $52 - 49 = 3$ (computational form)

3. Separate: Change Unknown. $3 + \boxed{4} = 7; 7 - 3 = \boxed{4}$ (computational).

4. Part-Part-Whole: Part Unknown. $3 + \boxed{5} = 8; 8 - 3 = \boxed{5}$ (computational form).

5. Separate: Initial Unknown. $\boxed{12} - 7 = 5; 5 + 7 = \boxed{12}$ (computational form).

6. Join: Change Unknown. $2 + \boxed{6} = 8; 8 - 2 = \boxed{6}$ (computational form).

7. Compare: Smaller Unknown. $5 - \boxed{2} = 3; 5 - 3 = \boxed{2}$.

8. Join: Result Unknown. $6 + 3 = \boxed{9}$ (computational form).

9. Part-Part-Whole: Whole Unknown. $8 + 5 = \boxed{13}$ (computational form).

10. Separate: Result Unknown. $12 - 4 = \boxed{8}$ (computational form).

11. Difference: Larger Unknown. $\boxed{13} - 8 = 5; 5 + 8 = \boxed{13}$ (computational form).

12. Join: Initial Unknown. $\boxed{7} + 4 = 11; 11 - 4 = \boxed{7}$ (computational form).