Exam 2 Study Sheet

Math 162
Prof.Pruim
Spring 2002

Exam 2 is scheduled for Friday, March 8, during class. The exam will cover Chapter 5, Chapter 6, Chapter 7, and Sections 8.1–8.3 (only center of mass part of 8.3), with an emphasis on newer topics. Most of the exam questions should be quite predictable. Here are some obvious things you should certainly be prepared for:

- Anti-derivatives/definite integrals

  This includes using the substitution method and integration by parts and the methods related to these, like trig substitution, partial fractions, etc. **You will not be allowed to use a calculator for this portion of the test since some calculators can evaluate integrals for you.**

- Applications of integration
  - area and volume, especially volumes of shapes formed by revolution and the area between curves
  - motion (displacement, velocity, acceleration)
  - work
  - average value of function
  - length and surface area
  - moments, center of mass, and centroids

- Understanding what integration is all about
  - relationship to area
  - Riemann sums
  - Approximation methods (left, right, midpoint, trapezoid, Simpson's rule)
    - The error bound formulas below will appear on the test, but you should know how to work with them.

Everything on the exam will fit under those categories, but here is some additional detail:

1. Terms you should be familiar with:
   - definition of logarithm in terms of an integral,
   - Riemann sum,
   - cross-section (including non-circular),
   - disk, washer, shell, cylinder in context of finding volumes,
   - work, Newtons, Joules, and foot-pounds, density of water (metric)
   - g, the gravitational constant
   - Hooke's law, and spring constant
   - the differential for arclength ds and its equivalent forms
   - moment about an axis (M_x and M_y)
2. Things you need to know
   - **Memorize the derivatives and anti-derivatives of our basic functions**: polynomials, powers, exponential and logarithmic functions (including "unnatural bases"), all six trig functions. (Don't forget $\int \sec x \, dx = \ln|\sec x + \tan x| + C$ and $\int \tan x \, dx = \ln|\sec x| + C$. (See table of integrals, numbers 1–15 in the back of your book.)
   - Be able to **recognize derivatives** when you see them. (Examples: $\sec x \tan x$, $\sec^2(x)$, $\cos(x)$, $\frac{1}{2\sqrt{x}}$, $\frac{1}{x}$, etc.)

- Know the most important **trig identities**. These include:
  - $\sin^2(x) + \cos^2(x) = 1$ (and its cousins, $1 + \tan^2(x) = \sec^2(x)$ and $\cot^2(x) + 1 = \csc^2(x)$),
  - the definitions of the 6 trig functions (triangles and unit circle forms),
  - $\sin^2(x) = \frac{1-\cos(2x)}{2}$
  - $\cos^2(x) = \frac{1-\sin(2x)}{2}$
  - You should be able to evaluate expressions like $\sin(\arctan(x/3))$ that arise when doing trig substitutions.
- Force = Mass $\times$ Acceleration,
- Work = Force $\times$ Distance,
- Hooke's Law for springs (force is proportional to displacement).
- Law of the Lever and how it applies to center of mass

3. Things you don't need to memorize.

   You should know how to use these, but you don't need to memorize them. They will be provided on the test if needed. (Or you will be expected to "figure them out" from scratch.)

   - $\int \sec^3(x)\, dx$ (But you should be able to derive it, see page 481.)
   - $\int \tan^3(x)\, dx$ (But you should be able to derive it, see page 481.)
   - $\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
   - $\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
   - $\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
   - Error formulas for approximate integration. The following will appear on the test, if needed:

     $$|E_T| \le \frac{K(b-a)^3}{12n^2} \qquad |E_M| \le \frac{K(b-a)^3}{24n^2} \qquad |E_S| \le \frac{K(b-a)^5}{180n^4}$$

     You need to know what they mean and how to use them.

4. Some advice

   - Show your work. This includes writing down the computations that you had your calculator do, not just the results. (I should be able to easily tell where each number came from.)
   - Practice, practice, practice the substitution method, including substitutions for trig integrals, integration by parts, partial fractions. You need to be **confident and quick** when working these methods.
   - **Look for simple things first.** Don't launch into complicated substitutions or parts until you have looked to see if something simpler is available. Leave "long" integration until the end of the exam, if you like.
   - **Read the directions.** I may ask you to do only part of a problem: for example, I could ask you to set up an integral, but not evaluate it. I may require you to show certain substeps clearly: set up form for partial fractions but don't find coefficients, find coefficients from partial fractions form, sketch a picture, write down a Riemann sum associated with an integral, or compute the volume of one "section" of a shape, etc.
   - **Be NEAT.** If you keep your work neat and organized, you are less likely to make errors and more able to correct them once made.
   - **Don't forget the differentials!** Don't leave off dx and its cousins in your integration.
   - I strongly recommend that you to **sketch a representative disk, washer, cylinder, or other cross-section** when finding volumes, arclength, surface area, moments, etc. I may even require it. It is very easy to make mistakes in finding radii or other dimensions. A sketch helps prevent these errors. Often just sketching the rectangle that generates the disk, washer, cylinder, cone, etc., is just as good.
   - **Use your time wisely.** Look over the exam before you begin and decide how to make the best use of your time. Don't spend lots of time not making progress.