Lab F: Working with Power Series

Today's lab consists of a number of problems for you to work on. Work in groups of two or three.

1. Use geometric series to find power series for the following functions and determine their intervals of convergence.

a) $f(x) = \dfrac{1}{1+x^3}$

b) $g(x) = \dfrac{1}{2+3x}$

c) $h(x) = \dfrac{x}{2+3x}$

2. a) Find a Taylor series for $f(x) = \ln(x)$ around the point $x = 1$.

b) Why was 1 chosen in this case rather than 0?

c) What is the radius of convergence for this series? (You don't have to check the endpoints this time, but you should know how to do so.)

d) Use your work above to find a power series for $g(x) = \dfrac{1}{x}$.

3. Consider the following power series representation of a function:

$$f(x) = \sum_{n=0}^{\infty} nx^n$$

a) Determine the radius of convergence for this power series.

b) Use the series to determine $f(0)$, $f'(0)$, $f''(0)$.

c) Can you figure out what function f is? (Hint: you can start from a geometric series.)

4. Let $h(x) = \int_0^x e^{-x^3}\, dx$.

a) Find a power series representation for h. (Begin with a power series for e^x.)

b) Use the first 3 non-zero terms your representation above to estimate $\int_0^2 h(x)\, dx$. How accurate is your approximation? Explain how you know.

c) What is the largest x for which this method (with 3 non-zero terms) yields an approximation that is within 0.001 of the correct answer?

5. Consider the following power series representation of a function:

$$f(x) = \sum_{n=0}^{\infty} n^2 x^n$$

a) Determine the radius of convergence for this power series.

b) Use the series to determine $f(0)$, $f'(0)$, $f''(0)$.

c) Can you figure out what function f is? (Hint: you can start from a geometric series.)

6. Let $g(x) = \dfrac{e^x - x - 1}{x^2}$

a) Find a Maclaurin series for $g(x)$. (Hint: start with a Maclaurin Series for e^x.

b) Use this Maclaurin series to determine $\lim_{x \to 0} g(x)$.