Lab E: Approximating Series

Some Sums (a.k.a. Series)

For each of the series below, find an approximation that is within the requested accuracy (specified by ε) of the correct value, or show that the series diverges. For convergent series, answers should be in the following form:

$$\boxed{\text{A}} \le \sum a_n \le \boxed{\text{B}}$$

where the boxes (A and B) are filled in to give both the numerical values and an indication of how you got them and the $0 \le B - A \le \varepsilon$ Note that by choosing the midpoint of the interval $[A, B]$ for an estimate, the error will be at $\varepsilon/2$. Of course, your goal is to use as few terms of the series as possible to get your estimates.

Notes:

- The first problem is done as an example. Use the format of that answer as a model for your own answers.
- You might like to type `?N` to find out how to get *Mathematica* to attempt to give you more digits in its numerical approximation.
- Whenever possible, avoid using `NSum[]` in this lab.

1. a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n}$ $(\varepsilon = 0.01)$

Answer: This is an alternating series, so $4.177377518 = \sum_{n=2}^{99} \frac{(-1)^n}{n} \le \sum_{n=2}^{\infty} \frac{(-1)^n}{n} \le \sum_{n=2}^{100} \frac{(-1)^n}{n} = 4.187377518$ A good estimate for the value of the sereis is 4.18238, it is within 0.005 of the correct answer.

b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n}$ $(\varepsilon = 0.005)$

2. a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ $(\varepsilon = 0.01)$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ $(\varepsilon = 0.001)$

3. a) $4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} + \cdots$ $(\varepsilon = 0.01)$.
b) $4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} + \cdots$ $(\varepsilon = 0.001)$.
c) What number do you think this is approximating?

4. a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ $(\varepsilon = 10^{-10})$ b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ $(\varepsilon = 10^{-20})$

c) What number do you think this is approximating? (Hint: look at recipricals.)
d) Check your work by using `N[]` to let *Mathematica* compute this number to 20 decimal places.

5. a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n}{3n+1}$ $(\varepsilon = 0.5)$. b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n}{3n+1}$ $(\varepsilon = 0.05)$.

6. a) $\displaystyle\sum_{n=2}^{\infty} \frac{(\ln n)^2}{n^2}$ $(\varepsilon = 0.05)$. b) $\displaystyle\sum_{n=2}^{\infty} \frac{(\ln n)^2}{n^2}$ $(\varepsilon = 0.01)$.

7. a) $\displaystyle\sum_{n=2}^{\infty} \frac{\sin^2(n)}{n^3}$ $(\varepsilon = 0.001)$. b) $\displaystyle\sum_{n=2}^{\infty} \frac{\sin^2(n)}{n^3}$ $(\varepsilon = 0.00001)$.