Lab B: Volumes

Determining the Volume of a Solid: an Example

Suppose you want to use *Mathematica* to help you with volume problems like the ones we have been doing in class the last couple days. Along the way to solving these types of problems, we did some of the following types of things:

- Sketch the region enclosed by some curves in the plane.
- Find intersection points of curves in the plane.
- Sketch a solid that results from rotating a region around an axis
- Set up an integral by considering Riemann sums. In particular, we needed to
  - decide which direction to slice,
  - decide what variables to use,
  - figure out how once rectangle contributes to the volume of the whole thing (disks, washers, cylinders, square slabs, etc.)
  - determine the endpoints of integration
- Evaluate the resulting integral.

Let's go through the steps above and see how *Mathematica* can help. As our first example, let's consider the largest finite region bounded by the curves with equation $y = \frac{5+4x^2-x^3}{10}$ and $y = \frac{5-2x+x^2}{10}$ and the volume that results when we rotate this region around the x-axis.

1. Define the functions $f(x) = \frac{5+4x^2-x^3}{10}$ and $g(x) = \frac{5-2x+x^2}{10}$ and plot them on the same axes. (`Plot[{f[x],g[x]},{x,-5,5}]`). Adjust the domain so that it clearly shows the region we are interested in.

*Mathematica*is capable of fancier graphs than we have seen so far. Before you can use these fancier graphs, however, you must enter the command:

```
Needs["Graphics‘"]
```

Be sure that you use the correct kind and number of quotation marks! Note that there is a reverse single quote after the word "Graphics", which must be capitalized.

2. Now plot the functions again using `FilledPlot[]` instead of `Plot[]`.

3. We need to know the intersection points of these two graphs. One point looks pretty obvious, the other not so much. Use `Solve[]` (or `NSolve[]` if `Solve[]` fails) to find the points of intersection. Then plot just the region we want using `FilledPlot[]`.

The volume we are interested in is a surface of revolution, and *Mathematica* even has a command built in to plot 3-D pictures of such things. Give this a try:

4. Use the command `SurfaceOfRevolution[g[x],{x,0,3}, RevolutionAxis->{1,0,0}]`.

5. Now try the following:

- `gsurf=SurfaceOfRevolution[g[x],{x,0,3}, RevolutionAxis->{1,0,0}]`.
- `fsurf=SurfaceOfRevolution[f[x],{x,0,3}, RevolutionAxis->{1,0,0}]`.
- `Show[fsurf,gsurf]`

After doing this, go back and fix the endpoints to show the region we really want.

6. Now experiment with the option `ViewPoint->{a,b,c}`. Put in 3 numbers for a, b, and c and it will be like looking at the shape from that point. Find the most interesting view you can.

By the way, to rotate things around the y-axis, use `RevolutionAxis->{0,0,1}`

7. Now set up the integrals and let *Mathematica* compute the volumes of the two shapes. Which is larger?

Some Additional Problems

8. A bowl is formed out of the bottom quarter of a sphere of radius r. Get *Mathematica* to make a plot of the bowl and determine how much liquid the bowl holds.

9. If you drill out the middle half of a sphere, how much volume is left? (That is, imagine drilling along a diameter with a bit that is one half the radius of the original sphere.) How much will have been removed?

You might like to guess before you start whether the removed part or the remaining part is larger.