Lab A: Integration in *Mathematica*

Review of *Mathematica*

A few reminders about *Mathematica*:

- *Mathematica* distinguishes between the return key and the enter key. Use return to go to the next line; use enter to execute a command.
- *Mathematica* distinguishes between three kinds of brackets
  - Round parentheses are used for algebraic grouping: `(x^2 + 1)(x-1)`
  - Square brackets are placed around the arguments to a function: `Sin[x]`
  - Curly brackets are placed around items in a list: `Plot[{Sin[x],Cos[x]},{x,-2*Pi, 2*Pi}]` will plot two trig functions on the interval $[-2\pi, 2\pi]$. *Mathematica* uses lists to describe intervals. The first item is the name of the variable, the next two are the endpoints.
  - Be careful using copy and paste in *Mathematica*. *Mathematica* distinguishes between several kinds of cells (input, output, text, etc.) and copying from one kind to another generally leads to problems. You may copy from one input cell to another, but do not copy from output or text cells and try to paste into an input cell.
- All built-in functions and constants are capitalized. Examples: `Sin[]`, `Cos[]`, `E`, `Pi`.
- You can get information about a function (built-in or your own), by using the question-mark operator. Example, to find out about the `Plot[]` function, type `?Plot` or `??Plot`. For built-in functions there may be a clickable link to additional information, examples, etc.
- You can define your own functions. Example: `f[x_]:=Sin[x]+E^x`. (The underscore is important; it indicates that x is to be treated as a variable.)

  It is a good idea to precede this with `Clear[f]` to remove any previous definitions, and to follow it with `?f` to check that it worked as you intended. It is a good idea to use functions definitions if the function you are using is at all complicated; it saves a good deal of typing and makes modifications or recovery from errors simpler.

With that review in mind. Let's get *Mathematica* to do some things for us. On the back side of this sheet are some problems for you to work on.

Some Integration Problems

1. Use *Mathematica*'s help to learn how `Integrate[]` and `NIntegrate[]` work. Use these to compute the following:

a) $\int \sin(x)\, dx$

b) $\int_0^\pi \sin(x)\, dx$

c) $\int \tan(x)\, dx$
Note that the answer you get doesn't look like the answer we got in class ($\int \tan(x) = \ln(|\sec(x)|) + C$). Plot both the result from class and the result from *Mathematica* and comment on the results.

d) $\int \sec(x)\, dx$ (We'll learn how to do this by hand in chapter 7.)

e) $\int \ln(x)\, dx$ (We'll learn how to do this by hand in chapter 7, too.)

f) $\int e^{-x^2}\, dx$ (Remember: Capital E.)
This can't be expressed using the functions we usually deal with. What does *Mathematica* do with it?

g) $\int_0^1 e^{-x^2}\, dx$

2. Now go back and do a few of the items above using *Mathematica*'s palettes. (If you were already using them, do some again without.) The palettes are a graphical way to enter you input. You can access *Mathematica*'s "Basic Palette" under the "File" menu.

3. Here is an integral neither you nor *Mathematica* can do alone. But together you can get it.

$$\int \frac{e^{2x}}{2+e^{3x}}$$

a) Let *Mathematica* give it a try.

b) Now try the substitution $u = e^x$. Work out by hand the resulting integral. (You will need to use $e^{2x} = e^x \cdot e^x$.)

c) Now let *Mathematica* do the integral you get after substitution. Voila!

d) Of course, we need to substitute back to the x variable. *Mathematica* can do this for you too. Append to the command you used to work out the integral with u in it the following: `/.u->x^2`. This tells *Mathematica* to substitute in x^2 wherever it sees u.

4. a) Is there a function f such that $f'(x) = x^2 + \sin(x)$ and $f(1) = f(-1)$? If so, find an example. If not, why not?

b) Is there a function g such that $g'(x) = \dfrac{1}{x^2}$ and $g(1) = g(-1)$? If so, find an example. If not, why not?

5. Consider the area enclosed by the curve $y = x^2$, the $x-$axis, the lines $x = 1$ and $x = 2$. What vertical line will divide this area into two equal parts?