Math 161, Fall 2001 Exam 4 Review Sheet

Exams in this class are always cumulative, but emphasize the most recent material. But since we are coming to the end of the semester, many of our recent topics build on things we have been doing all semester. Here are some of the important topics you should be sure to review.

Computing Derivatives

Of course, you will still need to know how to compute derivatives.

Meaning of Derivatives

This has been a major topic all semester. Especially important are the ways in which f, f' and f'' are related. Included now is also the idea of an *antiderivative.*

1. When units of y, x are given, what units does dy/dx have?
2. Meaning of derivative as a rate of change/growth (as a limit of average rates of change)
3. Higher derivatives
   - Notations for 2nd, 3rd, etc. derivatives: $\frac{d^2y}{dx^2}$, $\frac{d^3y}{dx^3}$, $\frac{d^ny}{dx^n}$, $f^{(n)}(x)$
   - $f''(x)$ is derivative of $f'(x)$, $f'''(x)$ (also written as $f^{(3)}(x)$) is derivative of $f''(x)$, etc.
4. What first and second derivatives tell us about the shape of a graph
   - Geometric interpretation (height, slope, concavity of graph)
   - Increasing, Decreasing, Concave up, Concave down
   - Critical Points, Inflection Points
   - First Derivative Test, Second Derivative Test
5. Interpretation in other settings:
   - Motion in time (position, velocity, acceleration, jerk)
   - Other rates of change (e.g., mass, linear density)

Optimization Problems

These problems build on what we learned prviously about local and absolute extrema, so you should review that materials as well.

1. Basic outline for applied problems and need for representation in terms of functions
2. Definitions of local extremum, absolute extremum, critical point, inflection point
3. Extreme Value Theorem
4. Finding local and absolute extrema
   - 1st and 2nd Derivative Tests
   - "Closed Interval Method" (see p. 282.) and its relation to the Extreme Value Theorem
5. Be sure to justify the claims you make in these problems.

Newton's Method

1. What it does (approximates roots of functions)
2. Big idea: a function can be approximated by its tangent line
   - How Newton's Method differs from linearization
3. Geometric intuition, derivation of iteration formula
4. How to use it to approximate numbers like $\sqrt{3}$
5. Situations where Newton's Method works well/poorly

Antiderivatives

1. Meaning of *antiderivative*
2. Finding antiderivatives (most general and spedcific) [like Jeopardy!]

Definite Integral

1. Definition using Riemann Sums
2. Approximation using Riemann Sums (including "Midpoint Rule")
3. Connection to areas, other applications
4. Calculating using Fundamental Theorem of Calculus

Readings

This test may include questions related to the readings and our discussion of them in class.

Miscellaneous

1. Use of trigonometric functions, especially in connection with optimization problems
2. Formulas for areas/volumes will be supplied *except* in following cases:
   - Circle: Area $= A = \pi r^2$, circumference $= 2\pi r$
   - Sphere: Volume $= V = \frac{4}{3}\pi r^3$, surface area $= 4\pi r^2$
   - Rectangle: $A =$ (length)(width)
   - Area of triangle: $A = \frac{1}{2}$(base)(height)
   - Volume of generalized cylinder: $V =$ (cross-sectional area)(height)
     Note: This includes formulas for volume of a right circular cylinder – $V = \pi r^2 h$ – and volume of a box.
   - Volume of a cone/pyramid: $V = \frac{1}{3}$(cross-sectional area)(height)
3. *Mathematica* There may be a question covering the use of *Mathematica*. Pay special attention to the *syntax* (capitalization, punctuation, etc.).