Math 161, Fall 2001
Test 2 Review Sheet

All tests in the class are cummulative, but will emphasize the most recent material. New sections covered on Test 2 include Appendix D (inverse trigonometric functions), 1.5–1.6, 3.1–3.6, and 3.8 This includes the following:

Inverse Functions

1. Definition of an inverse to f: a function g
   - whose domain is the range of f, whose range is the domain of f
   - that satisfies $f(g(x)) = x$ for each x in domain of g and $g(f(x)) = x$ for each x in domain of f
2. Invertibility
   (a) When a function is invertible (one-to-oneness, horizontal line test)
   (b) Restricting domain so a function becomes invertible
3. Graphs of inverses
   (a) If (x, y) on graph of f, then (y, x) is on graph of f^{-1}
   (b) Obtained by reflection through line $y = x$ from graph of f
4. Finding formula for f^{-1} when one for f given
5. Logarithms and exponentials ($\log_a x$ is inverse to a^x)
   (a) Properties of Exponenetial Functions (see page 58, for example)
   (b) Properties of Logarithms (see pages 68–70)
6. Inverse trig functions
   (a) How to restrict the domains
   (b) Working with unit circle and inverses

Computing Derivatives

1. Derivative Rules for Combinations of Functions
   (a) Constant-Multiple Rule (be able to prove)
   (b) Sum/Difference Rules (be able to prove)
   (c) Product/Quotient Rules (be able to prove)
   (d) Chain Rule (don't need to know the proof)
      i. Several forms in which it is written
      ii. How it is the basis for: implicit differentiation, derivatives of logarithmic functions, derivative formulas for inverse functions (including inverse trig functions), the generalized power rule
   (e) How to use combinations of the above
2. Derivatives of special functions
   (a) Derivatives of constant functions
   (b) Power Rule: $\frac{d}{dx}x^n = nx^{n-1}$, $n \neq 0$
   (c) Exponential functions: $\frac{d}{dx}a^x = (\ln a)a^x$

(d) Logarithmic functions: $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

(e) Trig. functions

i. Know them for all six trigonometric functions

ii. From the derivatives for $\sin x$, $\cos x$, be able to prove the other formulas

(f) Inverse trig. functions

i. Know derivatives of $\arcsin x$, $\arctan x$ (also written as $\sin^{-1} x$ and $\tan^{-1} x$)

ii. Know how to get formulas for inverse functions (including inverse trig functions) by implicit differentiation.

3. Implicit differentiation

   (a) How to carry it out to obtain dy/dx

   (b) In what situations would you want to use it?

4. Logarithmic differentiation

   (a) Idea behind it and how to carry it out

   (b) In what situations would you want to use it?

Interpretation of Derivatives

1. On a graph: slope of tangent line (i.e., steepness of curve)
2. In application settings: intantaneous rate of change
3. Relationship to average rates of change
4. When units of y, x are given, what units does dy/dx have?

Some Important Theorems

For each of the following you should know the "if part", the "then part", and where it was useful.

1. Differentiability Implies Continuity: If $f'(a)$ exists, then f is continuous at a.
2. Intermediate Value Theorem

Miscellaneous Items

1. What is meant by

   (a) $\sin^2 x$ vs. $\sin x^2$ (similarly for other trigonometric functions)

   (b) $\sin^{-1} x$ vs. $1/\sin x$ (similarly for any invertible function f)

2. As per the above, what is wrong with the statement $\dfrac{d}{dx} \tan^{-1} x \overset{?!?}{=} -\tan^{-2} x \sec^2 x$?

3. The number e

   - Defined as the number a for which $\lim\limits_{h \to 0} \dfrac{a^h - 1}{h} = 1$ (i.e., so $\frac{d}{dx}(e^x)|_{x=0} = 1$)

4. Some important limits and applications (see Exercises 35–44, p. 214) including

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \frac{\cos x - 1}{x} = 0$$

5. Difference between power functions (like x^3 or $x^{\sqrt{3}}$) and exponential functions (like 3^x or $3^{\sqrt{x}}$) and which differentiation rule applies to which.

6. How to get *Mathematica* to do derivatives (both ways)