Below are some problems for you to work on. Work in groups of two or three.

1. Suspended from the center of the ceiling of a rectangular room which is 9 meters by 12 meters is a rotating light which shines horizontally at the walls. The light rotates 5 times each minute, and as it does, a spot of light travels around the room along the walls.

a) Assuming that the light is shining directly at the center of one of the short walls at time $t = 0$ seconds, write a function $f(t)$ which gives the distance of the spot of light on the wall to the nearest corner at time t.

b) How fast is the spot moving when it is in the center of the long wall?

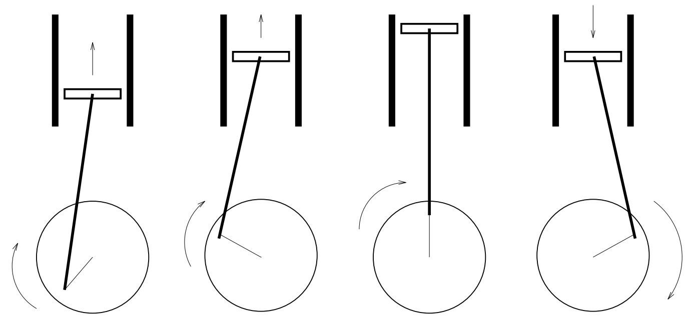
c) How fast is the spot moving when it is in the center of the short wall?

d) How fast is the spot moving is it is enters a corner going from a short wall to a long wall? How can we describe this quantity matematically?

e) How fast is the spot moving is it is leaves such a corner?

f) Where is the spot when it is moving most quickly?

2. Many devices turn circular motion into linear motion or vice versa by employing a version of the following device. One end of a rod is connected to a piston which is in a compartment which only allows linear motion. The other end of the rod is connected near the outside of a wheel which is free to rotate around its center. A diagram of such a system is pictured below at four different times. (This might be pistons and driveshaft in an automobile or the foot pedal and wheel of a spinning wheel or some similar manually powered device, for examples.)



In the following suppose the rod in the diagram above has length l and is attached to the wheel at a point R units from its center.

a) If the wheel spins at a constant rate and goes around twice every second, how fast is the piston moving when it is half way between its highest and lowest points?

b) When is the piston moving fastest?

c) Suppose $l = 8$ and $R = 2$, and that at time 0 the piston is at its highest point. Come up with a function which gives the velocity of the piston t seconds later. (What does the sign of your velocity indicate? How?)

d) Now come up with function which gives the position of the piston t seconds later. (What did you pick for height 0?)

3. For each of the following, determine whether they are true or false. If they are false find a counterexample and see if you can modify the statement in a meaningful way to make it true.

a) True or False: If $f'(a) = 0$, then f has a local extremum at $x = a$.

b) True or False: If f has a local extremum at $x = a$, then $f'(a) = 0$.

c) True or False: Every function which is one-to-one has an inverse function.

d) True or False: Every function whose derivative is never zero is a one-to-one function.

e) True or False: If $f(x)$ is a one-to-one function, then $f'(x)$ is never zero.

f) True or False: If $f(x)$ is a one-to-one function, then $f(x)$ has no local maxima or minima.

4. Sketch a crude graph of $y = g(x)$ if g has two continuous derivatives which satisfy the properties listed in the table below. Make sure that the horizontal axis of your graph is clearly labeled.

	$x < -2$	$x = -2$	$-2 < x < 0$	$x = 0$	$0 < x < 2$	$x = 2$	$2 < x < 4$	$x = 4$	$x > 4$
$g(x)$				-1					
$g'(x)$	$-$	-1	$-$	0	$+$	1		0	
$g''(x)$	$-$	0	$+$		$+$	0	$-$	0	$+$

5. Consider the equation $x^3 - 3x + q = 0$. What are the requirements on q for this equation to have exactly one solution? two solutions?? three solutions?? Sketch the graph of $y = x^3 - 3x + q$ in each case.