In this lab you will be introduced to some of the numerical and algebraic things *Mathematica* can do. You will also be reviewing how to work with *Mathematica* expressions, and how to define and plot functions. Work in groups of two at one computer and turn in one set of solutions to the exercises for your group (with both names on it).
Remember, you can select all the things you want to print by clicking on the bars at the right side of the notebook. Hold down the apple key while you do this to add each new item to your selection. Then when you choose "Print Selection" from the "File" menu, only what you have selected will print.

# 1 Some Review

**1.** a) Enter `Sin[Pi]`, `N[Sin[Pi]]`, and `Sin[N[Pi]]`. Record the results and explain the differences you see.

b) Which is larger: $e^\pi$ or $\pi^e$? About how much larger?

# 2 Options to Plot

If you type `??Plot` (try it now), you will see a list of options to the plot command. These options allow you to control various aspects of your graphs like colors, line thickness, etc. We will learn about two useful options. `AspectRatio->Automatic` will make the two axes use the same scale. This is important if you want to see circles as circles (rather than ellipses) or to "see" the slope of a line without having to adjust for the scales.
Compare the results of the following two commands:

- `Plot[Sqrt[4-x^2], {x,-3,3}]`
- `Plot[Sqrt[4-x^2], {x,-3,3}, AspectRatio->Automatic]`

**2.** You will get some error messages when you do the example above. Why do you get error messages and what can you do to eliminate them?

You can set the color of the graph, use `PlotStyle -> Hue[]`. For example, `Plot[Sqrt[4 - x^2] , {x, -3, 3}, PlotStyle->Hue[.5]]` or `Plot[Sqrt[4 - x^2] , {x, -3, 3}, PlotStyle->Hue[1 1 0]]` . See `??Hue` for more information.

# 3 Plotting Multiple Functions

Often you want to see more than one function on the same coordinate system. `Plot` is capable of doing that. Type `?Plot` and see if you can figure out how to do it. (You might want to click on the word "More" to get additional help and examples if you can't figure it out from the first results you get from `?Plot`.

**3.** Plot three functions on a single coordinate system: $f(x) = \sin(x)$, a graph that is shifted *left* 2 units, and a graph that is compressed by a factor of 3 horizontally and stretched by a factor of 2 vertically (it will wobble 3 times as fast and 2 times as high). Print out both the graph and the command you used to generate it (but not the rest of your notebook). You might like to make them different colors, but this is not required. This is the only thing you must print.

Plotting multiple functions makes use of an important *Mathematica* structure: a list. Lists in *Mathematica* are surrounded by curly braces. May *Mathematica* functions like `Plot[]` can handle lists of inputs as well as single inputs. For example, to get the value of a function at several points, one can type something like `f[{1,2,3,4,5}]`. Define a function and give it a try to see what the output looks like. You could also use this to compare two values: `{Sin[1],Sin[2]}`.

# 4 Algebra and Solving Equations

*Mathematica* has several methods of manipulating expressions algebraically and "solving" equations. We will learn about some of these today. Enter the following and see what they do:

- `Expand[(x+1)^8]`
- `Factor[-16 +26x -11 x^2 + x^3]`

  Notes:
  - By default, `Factor[]` only factors with rational coefficients.
  - You can factor integers using `FactorInteger[]`:
- `Expand[%]` (% stands for the result of the last operation, a handy short-cut.)
- `Simplify[(x-a) (x+a) + a^2]` (Always pays to simplify.)
- `Apart[2/ (a^2 - 4a + 3)]`
- `Together[a + b/x + c/x^2]`

**4.** Based on the results above, write a brief description of what each of these five functions does.

**5.** Find the factors of 123456789 and 987654321. (You may need to read some of the help information or experiment a little bit to figure out how to interpret the results you get.)

`Solve []` and `NSolve[]` can be used to sove equations. `Solve[]` attempts to find exact solutions by algebraic means (by factoring, rearranging, etc), and `NSolve[]` attempts to find numerical solutions. Type `Solve[ x^2 -x -1 == 0, x]`. Notice that *Mathematica* uses `==` to represent an equals sign in equations. The last `x` tells *Mathematica* what variable to solve for.
*Mathematica*can even solve equations involving trig functions:

**6.** a) Use *Mathematica* to solve $\sin(x) = \cos(2x)$.

b) *Mathematica* will warn you that it may not have found all solutions. Find at least one solution *Mathematica* missed, or explain how you know they have all been found.

Now try solving $\sin(x) = 5\cos(x)$. This time the answer is expressed in terms of the functions $\arccos(x)$. We will learn about that function later this semester. For now, try using `NSolve[]` in place of `Solve[]`. `NSolve[]` gives its results as decimal approximations.

**7.** Use `Solve[]` and `NSolve[]` to find all the roots of the function $g(x) = \dfrac{x^7 - 6x^5 + 11x^3 - 6x}{1 + x^2}$ Give exact values for as many as you can. Give decimal approximations for all solutions.

**8.** a) Give numerical approximations to the solutions of $\sin(x) = 5\cos(x)$.

b) *Mathematica*again warns that some solutions may not be included in its list. Find a missing solution or explain why no more solutions exist.

**9.** Find all real solutions to $x^7 - x^6 - x^5 - 4x^3 + 3x^2 + 5x + 1$. Give exact values for as many as you can. Give decimal approximations for all solutions.