In this lab you will be introduced to *Mathematica*. You will learn how to work with *Mathematica* expressions, and how to define and plot functions. Work in groups of two at one computer and turn in out set of solutions to the exercises for your group (with both names on it).

1 An Introduction to *Mathematica* Functions

We have mentioned several times already the importance of functions for calculus. *Mathematica* is a computer program for doing mathematics and functions play a very important role in *Mathematica*, too.

1.1 Built-in Functions

Of course, *Mathematica* knows about some of the most common mathematical functions, like $\sin(x)$, $\cos(x)$, $\ln(x)$, $\sqrt{x}$, e^x, and many more. When using *Mathematica*'s built-in functions there 2 very important things to remember:

- All built-in functions are spelled with the **first letter CAPITALIZED**.
- The inputs to *Mathematica* functions are placed inside **SQUARE BRACKETS**.

So, for example, if you want to know the square root of 19, you can type `Sqrt[19]`. `Sqrt` is the *Mathematica* name for the square root function (notice the capital S), and the input (19) is placed inside square brackets.

1.2 Built-in Constants

*Mathematica*also knows a number of constants like π (`Pi`) and e (`E`). It even knows about infinity (`Infinity`) and the complex number i (`I`). Notice that **built-in constants are also capitalized.**

1.3 Arithmetic Operations

The arithmetic operations in *Mathematica* are probably similar to what you have used on a graphing calculator:

- `+, -, *, /` are used for addition, subtraction, multiplication and division.
- Round parentheses – `(` and `)` – are used for grouping.
- The caret symbol – `^` – is used for exponents
- Parenthesis and the multiplication symbol (`*`) can be omitted in the same situations were we omit them in mathematics, but it never hurts to use them just to make sure *Mathematica* knows just what you mean.

Examples

Try these examples. Note that you should see a horizontal line across the notebook before you begin (if not, click on the notebook) and that you must type **ENTER** (not return) to get a cell to execute. (Shift-return works, too.)

Math	*Mathematica*
$\sqrt{4+5\pi}$	`Sqrt[4 + 5 Pi]`
$\sin(\pi/3)$	`Sin[Pi / 3]`
$\ln(e^2+e)$	`Log[E^2 + E]`

Exercises

1. Copy down exactly (with pencil on paper) the output *Mathematica* gives to each of the examples above. Is the output what you expected? In just a moment you will learn how to get the output you probably expected.

2. Copy down exactly (with pencil on paper) the result *Mathematica* gives when evaluating the following: a) $\cos(\pi^2)$, b) $\left(\frac{3}{4}\right)^5$, c) $(0.75)^5$.

Note that the last two answers are different. Why do you think this is?

2 Some Special Functions

2.1 `N[]`

When it is able to, *Mathematica* will generally give *exact* answers rather than *approximate* answers. If you want a decimal approximation and *Mathematica* has given you an exact value, you can use a built-in function to request an exact value from *Mathematica*. The name of this function is `N[]`. For example, to get an approximation for $\frac{55}{34}$, enter `N[55/34]`. Try it.
If you get an exact value and want a decimal approximation, you can use a short-cut: `N[%]` will give the numerical approximation to the last output. (`%` always stands for the last output in *Mathematica*.)

Exercise

3. Use `N[]` to get decimal approximations for each of the items in problem 1. (Hint: you can go back and edit the input cells you entered there; that saves some retyping.)

2.2 ?

The question mark can be used to get information about functions. If you type `?N`, for example, *Mathematica* will tell you something about the function `N[]`. `??N` will give somewhat different information. And `?N*` will give you a list of *Mathematica* functions that begin with the letter N. If none of these methods give you the information you need, you can also look in the extensive online help menus.

Exercises

4. What is the fourth function in the list of *Mathematica* functions beginning with the letter A? What does it do?

5. Use the information from `?N` and `??N` to determine the 496th through 500th digits of π (after the decimal point).

6. The list of the Fibonacci numbers begins $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89\ldots$ Each term is the sum of the preceding two. These numbers have many interesting properties.

a) Find numerical approximations to the quotients of consecutive terms a) $\frac{8}{5}$, b) $\frac{13}{8}$, c) $\frac{21}{13}$, d) $\frac{34}{21}$, and e) $\frac{55}{34}$

b) What do you notice about these numbers?

c) Does *Mathematica* have a built-in function to compute Fibonacci numbers? Check by entering `?Fib*` to see if any functions begin with those letters. If there is such a function, compute the ratio of the 26th Fibonacci number divided by the 25th Fibonacci number. How does this ratio compare to the ones above?

2.3 `Plot[]`

To see the graph of a function, use the *Mathematica* function `Plot[]`. Here is an example:

```
Plot[Sin[x],{x, -2*Pi, 2*Pi}]
```

Type it in and see what you get. Remember to hit "Enter", not "return".
`Plot[]` is considerably more complicated than any function we have seen so far. It can take several inputs, but the basic form is like the example above:

- The first input provides an expression to plot – `Sin[x]` in our example.
- The second input tells what variable to use for the input to the function and what portion of the graph to display. In our example, the input variable is `x` and the the plot includes all values of x between -2π and 2π.

There are a number of other options to the `Plot[]` command controlling things like colors, thickness of lines, aspect ratio, range of output values to display (in case you don't like the choice *Mathematica* makes for you), etc. We will introduce some of these later as we need them. Consult your *Mathematica* Quick Reference Sheet, `??Plot`, or the help menus to learn more.

Exercise

7. Make a plot of the function defined by the equation $f(x) = x^2 + 3x - 1$ on the interval $[-3, 3]$. Click on the bar to the right of the graph. This selects that cell of the notebook. Now print out *just this graph* by using "Print Selection" from the print menu.

3 Defining Your Own Functions

You can also define your own functions in *Mathematica*. This is one of the most important things to learn to do correctly. The syntax is not difficult, but must be followed precisely to avoid errors. Read the section from your Quick Reference on defining functions. Notice especially

- **Always clear the function before defining it.** This will avoid bad interactions with previous definitions.
- **An underscore must follow the input variable.** This is how *Mathematica* identifies the input variable.
- As always with *Mathematica* functions, **be sure to use square brackets.**
- Usually you will want to **use "colon-equals"** (`:=`) between as shown to define a function. We won't discuss here the other kinds of equals signs in *Mathematica*, but **don't use another kind of equals sign unless you are sure you know what that would mean.**
- You do not need to use 1-letter names. You don't need to capitalize (In fact, it is a good idea not to for your functions; that way you can tell yours from *Mathematica*'s.) You do not need to use `x` as the input variable. It is perfectly acceptable to do something like

```
area[r_] := Pi * r^2
```

- If you have trouble with a function, try the following **two diagnostics** first:
  a) Type `?`*functionname* to see what *Mathematica* is thinking about your functions.
  b) Enter a particular value (e.g., `f[1]`) and see if the results are reasonable

Exercises

8. Only the last item of this problem requires you to turn anything in, but you will need to do the other parts to get to that point.

a) Define the function $g(x) = 3x^2 - x^6$.

b) Type `?g` to see what *Mathematica* tells you about g. Check to make sure everything is in order.

c) Have *Mathematica* compute $g(1.1)$ and $g(11/10)$. How do the answers compare?

d) Plot $g(x)$ on the interval $[-1, 2]$. Hint: use

```
Plot[g[x], {x,-1,2}]
```

Be sure you understand how this works.

e) Click on the graph. Your cursor should change to the graphics cursor. Now hold down the "apple key" as you move the mouse around on the graph; the coordinates for the point the cursor is over are displayed at the bottom of the screen. Use this to approximate when $g(x) = 0$. We will learn better ways to have *Mathematica* help us determine when a function has a zero-value in a future lab.

9. Now let $g(x) = \dfrac{x^7 - 6x^5 + 11x^3 - 6x}{1 + x^2}$. (Be sure to use `Clear[]` to get rid of the old definition of g.) Use *Mathematica* plots to estimate the values of x for which $g(x) = 0$. Such a value is called a root of the function. Zoom in until you can estimate each root to within 0.001.

Hints: Try an initial interval of $[-10, 10]$. There are 7 roots of this function. In order to get within 0.001, you will need to zoom in on some of the roots. Others you can probably just guess and check. For example, it looks like 1 is a root. Enter `g[1]` to check if this is indeed the case.