

All tests are cumulative, but the topics below are the primary focus of this test.

### Interpretation of functions and their derivatives/antiderivatives

1. Geometrically (height, slope, concavity of graph)
2. Motion in time (position, velocity, acceleration, jerk)
3. Other rates of change (e.g., mass, linear density)

### Applications of derivatives

1. L'Hospital's rule for determining limits
  - (a) Indeterminant forms to which L'Hospital's rule applies:  $\frac{0}{0}$ ,  $\pm\frac{\infty}{\infty}$
  - (b) Other indeterminant forms ( $0 \cdot (\pm\infty)$ ,  $\infty - \infty$ ,  $0^0$ ,  $\infty^0$ ,  $1^\infty$ ) and algebraic tricks to make L'Hospital's rule applicable
2. Linearization and Differentials
  - (a) Purpose: for approximating the *output* value of a function at point
  - (b) Big idea: estimating function with tangent line.
  - (c) Differentials of functions and their use in determining maximum error
  - (d) how to approximate numbers like  $\sqrt[4]{17}$
3. Newton's Method
  - (a) Purpose: to find approximate solutions (i.e., *inputs* to  $f$ ) to equations of form  $f(x) = 0$  (what kind of  $f$ ?)
  - (b) The formula  $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$ , and
    - i. how it was derived (geometric picture, following tangent line, etc.)
    - ii. how to compute iterates quickly on your calculator.
  - (c) How to use it to approximate numbers (like  $\sqrt[4]{17}$ )
  - (d) Drawbacks (might not converge, might converge slowly, might converge to a different root than you want) and how to deal with them
  - (e) Demonstrating graphically what happens on each iteration of the method
4. Local and Absolute extrema
  - (a) Definitions of local extremum, absolute extremum, critical point, inflection point
  - (b) Extreme Value Theorem
  - (c) Finding local and absolute extrema
    - 1st and 2nd Derivative Tests
    - "Closed Interval Method" (see p. 282.) and its relation to the Extreme Value Theorem

## 5. Applied optimization problems

### (a) General outline for solving:

- Express quantity to be optimized as a function of one variable. (If you get stuck here, ask for help so you can continue.)
  - This may require manipulating equations with several variables along the way.
  - This is where specific knowledge of the application area enters in.
- Determine appropriate domain for function you just found
- Apply techniques for finding extrema

### (b) Formulas for areas/volumes will be supplied *except* in following cases:

- Circle: Area =  $A = \pi r^2$ , circumference =  $2\pi r$
- Rectangle:  $A = (\text{length})(\text{width})$
- Area of triangle:  $A = \frac{1}{2}(\text{base})(\text{height})$
- Volume of generalized cylinder:  $V = (\text{cross-sectional area})(\text{height})$   
(Note: This includes formulas for volume of a right circular cylinder  $V = \pi r^2 h$  and volume of a box.)

## Antidifferentiation and Integration

### 1. Antiderivatives

- (a) Determining for nice functions
- (b) Notation using indefinite integral
- (c) More than one antiderivative for a given function (How are they related?)
- (d) Verifying that an antiderivative is correct using differentiation

### 2. Definite Integral

- (a) Definition in terms of limits of Riemann sums
  - i. Careful statement, including notation and explanation of what  $x_i^*$  and  $\Delta x$  mean. (see page 278)
  - ii. Geometric interpretation
  - iii. Using this to approximate an integral (left sums, right sums, “mid-point rule”, etc.)
  - iv. Situations in which we can know whether the estimate is too high or too low.
- (b) Basic properties (see pages 385-387)

### 3. Fundamental Theorem of Calculus

- (a) Complete Statement (see page 397)
- (b) Use in determining derivatives of functions involving definite integrals (1st part)
- (c) Use in evaluating definite integrals (2nd part)

### 4. Applications

- (a) Area under curve
- (b) Total Change Theorem
- (c) Relationship to Riemann sums