Math 161 D
Fall 2000

Lab A: Covered Wagon on a Bumpy Road

You may work in groups of 2 or 3 for this lab.

Starting from scratch in *Mathematica*

In each of our previous labs, you have begun by opening a *Mathematica* Notebook. This time we will start "from scratch." To begin *Mathematica*,

1. Click on **Lab n** in the upper right corner.
2. Find the *Mathematica* icon and double click on it to launch the program.

Using `ImplicitPlot[]`

In this lab we will use *Mathematica* and implicit differentiation to explore an interesting graph. *Mathematica* will produce graphs of equations using an `ImplicitPlot[]` function. In order to use this function, you must load the package that contains it by typing the following:

```
Needs["Graphics`"]
```

It is very important that you get the syntax correct for this command. In particular, notice the surrounding double quotes, the reversed single quote, the capitalization and the lack of any spaces. **Before continuing** check that this has been done correctly by typing

```
?ImplicitPlot
```

If the package has loaded properly, you should get information about the syntax of the function and what it does. Once the package has been properly loaded (but not before), try typing

```
ImplicitPlot[x^2 + y^2 == 1, {x,-2,2}]
```

This should plot a circle for you.

The basic form of `ImplicitPlot[]` is

`ImplicitPlot[` *equation* `, {x,a,b}]`

- Other options (like `PlotRange->{a,b}`, or `AspectRatio->Automatic` can also be added and will be useful below.
- Remember to use `==` in your equation.
- For more information, consult the output of `?ImplicitPlot` or `??ImplicitPlot`.

An interesting curve

The curve we want to consider is the graph of

$$2y^3 + y^2 - y^5 = x^4 - 2x^3 + x^2 \ .$$

1. Have *Mathematica* graph the curve and print 2 large copies of it.

Hints:

- Define functions $f(x) = x^4 - 2x^3 + x^2$ and $g(y) = 2y^3 + y^2 - y^5$. This will save you a great deal of effort.
- Just to be safe, use `Clear[f,g]` before you define f and g.
- You may need to adjust the x and y ranges a bit to get a good picture. You should see something that fits the title of this activity.
- If you select the graph and drag on it you can make it larger. It will print roughly as large as you see it on the screen.
- You can use "Print selection" to print only the graph and not your entire notebook.

Your report will consist of these two graphs and some things you write on them in problems 2–3 below. (Simply handwrite the answers to the questions on your graphs.) You should all do problems 1–3. Problem 4 is for those of you get through problems 1–3 more quickly, but it is not required and is not part of the report.

2. How many locations on the graph have horizontal tangents? Determine the coordinates of each of theses points using the following method:

a) Use implicit differentiation to find $\frac{dy}{dx}$. Start from

$$g(y) = f(x)$$

and show that $\frac{dy}{dx} = \frac{f'(x)}{g'(y)}$. (Note: this will be true whenever a curve is described by $g(y) = f(x)$.)

b) Let *Mathematica* help you determine when $\frac{dy}{dx} = 0$. (Use both `Solve[]` and `NSolve[]` to get both exact values and decimal approximations.

Hint: You can use `f'[x]` instead of `D[f[x],x]` to get $\frac{df}{dx}$.

c) Determine both coordinates of each point with a horizontal tangent and **label them on your first graph**.

d) How would you find points with vertical tangents? (Explain in a sentence or two, but do not actually do it.)

3. The most interesting points on the graph are the places where the graph crosses itself at $(0, 0)$ and $(1, 0)$.

a) What happens if you try to get the slope of the tangent line at these points using the expression for $\frac{dy}{dx}$ that you computed above? Why do you think this is?

b) Estimate the slope of the tangent lines at the points $(0, 0)$ and $(0, 1)$. Make use of *Mathematica*'s graphing and/or algebraic power.

Hint: You may find the `AspectRatio->Automatic` and `PlotRange->{a,b}` options handy.

c) Sketch these tangent lines on your second graph and label their slopes.

4. If you still have time, find all the points on the graph such that $x = 3/2$. (You will probably need to use `NSolve[]` rather than `Solve[]`.) Make a third graph, label each of these points, sketch the tangent lines at those points, and label the slopes of the tangent lines.