Math 161 D
Fall 2000

Lab 7: Differentiation

You may work in groups of 2 or 3 for this lab.

Differentiation in *Mathematica*

The only part of your lab manual (and the *Mathematica* notebook that you need for today's lab is pages 61 and 62 (from "Using the Differentiation Operator D" through Exercise 3). Begin by working through that material. If you can do Exercise 3, you have learned enough for now. This will explain how to get *Mathematica* to do differentiation. You do not need to include a record of Exercise 3 in your report.

Some Problems to Do

For the remainder of the period, work on the following problems. You may use *Mathematica* whenever it is helpful. You might like to think about how to get *Mathematica* to help you, even if you know how to do things by hand or with a calculator already. I've included some *Mathematica* hints along the way, too.

Your **report** should be a careful write up of solutions to as many of these as you get. Take notes as you work onthe problems and then have different members of your groups write solutions to different problems. Do the first three problems in order. After that, you may choose which problems you want to work on.

1. This one requires *Mathematica*. *Mathematica* knows all the rules you know (and then some) for differentiation. For example, type `D[g[x]*h[x],x]` and see what you get. (Clear g and h first if you have already defined them. Note that the square brackets tell *Mathematica* that g and h are functions of x, even though they have not been defined. What happens in you leave off the `[x]`? Why?)

a) Get *Mathematica* to give you the Quotient Rule and Chain Rule.

b) For the quotient rule, you probably got something that looks a little different from the form we wrote in class. Use `Together[]` to force *Mathematica* to express things as one fraction.
Here is an example of the syntax: `Together[7/x + 10/(x * x)]` will result in $\frac{10+7x}{x^2}$. You can also type `// Together` at the end of a line to make the output on that line be "`Together`'ed".

`Together[]` is much like `Simplify` in that it changes the form of an expression but not its value. Algebraic manipulations *Mathematica* can do include

`Simplify[]`	tries to get a "simple" expression
`Together[]`	puts over common denominator
`Cancel[]`	tries to remove common terms from fractions
`Factor[]`	tries to express things as products of terms
`Expand[]`	perfroms multiplications to write as a sum of terms
`Apart[]`	tries to get simple denominators

2. It is possible to take derivatives of derivatives. For example, if $f(x) = x^3$, then $f'(x) = \frac{df}{dx} = 3x^2$, and the second derivative is $f''(x) = \frac{d^2 f}{dx^2} = \frac{d}{dx}\left(3x^2\right) = 6x$. Of course, the third derivative is $f'''(x) = \frac{d^3 f}{dx^3} = 6$. And so on.

Let $g(x) = \sin(x)$. Determine $g'(x)$, $g''(x)$, $g'''(x)$, and $g''''(x)$. What is $g^{(59)}(x)$ (the 59th derivative of g)? How do you know?

3. Let $f(x) = x^3 + 3x^2 + 3x + 5$.

a) Does f have an inverse? How do you know? How could a derivative and the Intermediate Value Theorem be useful in figuring this out?

b) If f has an inverse, determine $f^{-1}(5)$, $\left(f^{-1}\right)'(5)$, $f^{-1}(12)$, and $\left(f^{-1}\right)'(12)$. Do this without getting an explicit formula for f^{-1}. (We have not learned how to compute derivatives of inverses yet, but you should be able to figure this out from the geometry. A `Plot[]` or hand-drawn sketch might be useful.)

c) Now get an explicit formula for f^{-1} and check your work above.

4. a) Let $F(x) = g(x)h(x)$. Compute F', F'', F''', and $F''''(x)$ in terms of g, h, and their derivatives. (*Mathematica* hint: `D[f[x],{x,n}]` will give the nth derivative.)

b) Compute $(A+B)^n$ for $n = 1$, 2, 3, and 4. (*Mathematica* hint: Use `Expand[]` to get cite Mathematica to expand $(A+B)^n$ for you.

c) Compare the results above. What do you notice?

5. Find all points on the parabola with the equation $y = x^2 - x$ such that the normal line at the point goes through the origin. (A normal line is perpendicular to the tangent line.)

6. Assume that $f(x)$ is a differentiable function, and that the values of f and its derivatives at the points $x = 0$, 1, 2, and 3 are given as follows:

$f(0) = 3$	$f(1) = 5$	$f(2) = -2$	$f(3) = 6$
$f'(0) = -1$	$f'(1) = 0$	$f'(2) = 3$	$f'(3) = 1$

Let $g(x) = x^2 - 3x + 2$.

Calculate $\dfrac{d}{dx}\big(f(x)g(x)\big)$ at $x = 0$, 1, 2, and 3.

7. Let f be the function such that $f(x)$ is the value of the sine of an angle which measures x **degrees**. (NOTE: For most values of x, $f(x) \neq \sin x$. Why not?) Let g be defined similarly for cosine.

a) Express f and g in terms of sin and cos.

b) What is $\dfrac{df}{dx}$? What is $\dfrac{dg}{dx}$? (HINT: Use part a) and the chain rule.)

c) Express $\dfrac{df}{dx}$ and $\dfrac{dg}{dx}$. in terms of $f(x)$ and $g(x)$. (No mention of sin or cos allowed.)

d) Is it still true that $(f(x))^2 + (g(x))^2 = 1$?

e) Why don't we use degrees in calculus?

8. The mathematicians at Los Alamos Laboratory developed the following equation to describe the change over time in the number of people infected with the AID's virus.

$$\frac{dI}{dt} = \alpha I(t)\left[1 - \frac{I(t)}{N}\right]$$

where

$$\begin{aligned} I(t) &= \text{number of people infected at time } t \\ N &= \text{size of the population} \\ \alpha &= \text{rate at which an infected person passes on the virus per unit time.} \end{aligned}$$

Assume t is measured in days, $N = 100{,}000$, and $\alpha = 0.01$. Answer the following questions according to the model.

a) Is the number of infected persons increasing or decreasing?

b) How many people have to be infected for the number of people infected to stop increasing?

c) How many people are being infected per day when there are 100 people infected?