

Test 3

Be sure to show your work on all problems. Unsubstantiated answers will not receive full credit.

1. Let $A = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 5 & 1 \end{bmatrix}$.

- a) How many rows does C have? How many columns?

- b) Is $A \cdot C$ a legal matrix operation? If so, what is the shape (number of rows and columns) of the resulting matrix. If not, explain why not. (**Do not actually compute the product.**)

- c) Is $C \cdot A$ a legal matrix operation? If so, what is the shape (number of rows and columns) of the resulting matrix. If not, explain why not. (**Do not actually compute the product.**)

- d) Verify that A and B are inverses.

- e) If there is a matrix X such that $AX = C$, what shape must X have (how many rows, how many columns)?

- f) Find a matrix X such that $AX = C$.

2. Let G be the graph represented by the matrix

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & \boxed{0} \\ 1 & 0 & \boxed{0} & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

- a) What kind of graph is G (directed, undirected, multigraph, etc.)?
- b) How many vertices does G have?
- c) How many edges does G have?
- d) What are the total degrees of each vertex?
- e) Compute the **top row** and **left-most column** of M^2 (using ordinary high school arithmetic). Explain what these values tell you about the graph G .
- f) Write down the next two matrices that arise when doing Warshall's algorithm on this graph. Circle the entries in these graphs corresponding to the boxed entries in M above and explain what they tell you about the graph G .
- g) Give good a big O estimate for the running time of Warshall's algorithm on a graph with n vertices and e edges. Briefly explain how you arrived at your estimate.

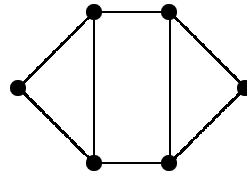
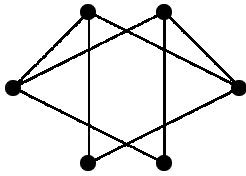
3. a) If you roll four 4-sided dice (each with four sides numbered 1, 2, 3, 4), what is the probability that the four numbers rolled will all be the same?
- b) If you roll four dice (each with six sides numbered 1, 2, 3, 4), what is the probability that the three numbers rolled will be distinct? (no repetitions)
4. Consider the following game. A player rolls two four-sided dice and wins the difference between the two values. (So a roll of two 2's wins nothing, but a roll of one 2 and one 4 wins \$2.)
- a) What is the probability that the player wins some money on a particular roll?
- b) What is the probability that the player wins nothing on a particular roll?
- c) What is the expected winnings per play (expected value of the random variable describing winnings)?
- d) If you have to pay \$1 for each roll to play, would you expect to make money or lose money in the long run? How much? (Note: if you pay \$1 to play and then roll (1,2), you break even.)

5. An undirected multi-graph is to have 10 vertices labeled 0 through 9. The degree of each vertex is to be the same as its label. Is there such a graph? If so draw or otherwise describe one. If not, explain how you know there is none.

6. a) Carefully describe how to tell if a *directed* multigraph has an Euler cycle.

b) Illustrate by drawing two directed multi-graphs, one with an Euler cycle and one without. Each graph should be connected and have between 6 and 8 vertices.

7. Are the following two graphs isomorphic? Explain how you know.



8. Consider the following pseudo-code:

```
input a graph G and a vertex v in G;
let Q be an empty queue;
mark v as visited;
put v into Q;

while Q is not empty{
  remove v from front of Q
  for each vertex w that is adjacent to v {
    if (w is unmarked) {
      mark w as visited
      put w into Q
    }
  }
}
for each vertex v of G {
  if v is unmarked return 0;
}
return 1;
```

a) This pseudo-code computes a boolean function (it returns either 0 or 1). What function is it computing?

b) If G is represented as an adjacency **matrix**, give a good big O estimate for the running time.

c) If G is represented as an **adjacency list**, give a good big O estimate for the running time.

d) For what kinds of graphs is an adjacency list most likely to be more efficient representation (for this and many other algorithms) than a matrix?