

## Solutions

**4** I LOVE MATH = L ORYH PDWK. Just move three letters toward the end of the alphabet to encode, wrapping to the beginning of the alphabet as needed.

**5** Here are the solutions using keys of 3, 6, and 9 to encode:

- HELLO: KHOOR, NKRRU, QNUUX
- GO KNIGHTS: JR NQLJKWV, MU QTOMNZY, PX TWRPQCB
- TO INFINITY AND BEYOND: WR LQILQLWB DQG EHRQG, CX RWORWRCH JWM KN-HXWM, ZU OTLOTOZE GTJ HKEUTJ,

**6** CORRECT, YOU GOT IT, ITISGOOD, HARDER WTHOUT KEY ( $e = 9$ )

**7** subtract 3, 7, or 9. That is, move in the reverse direction through the alphabet by the amount specified by the key.

**8** For a key of  $e$  we can decode by adding  $26 - e$  since this is the same thing (mod 26) as subtracting  $e$ .

**9** A and N are both coded as A (0), B and O are both coded as C (2). This means that when we see, for example, the letter C in the encoded message, we can't tell if the original message had a B or an O.

**10** Multiplying each number from 0–25 by 3 we get a different number each time. This means it is at theoretically possible to recover the original message without any guesswork.

**11**  $e = 4$  has the same problems that  $e = 2$  had.

**12** The encoding key cannot share any divisors (factors) with  $26 = 2 \times 13$ . So the encoding key may not be even, or 13. Other encoding keys will work.

**13** Q DANANA, FVYC OQZEC SZMAF, MF FI DZIFM, KIHFYHYGAFYQN ZQGEC

**14** DECODE WITH NINE (use  $d = 9$ )

**15**  $5 \cdot 21 = 105 \equiv 1 \pmod{26}$ , so 21 is the decoding key.

**16**  $2^5 = 32 \equiv 32 \pmod{65}$ ,  $3^5 = 243 \equiv 48 \pmod{65}$ ,  $4^5 = 1024 \equiv 49 \pmod{65}$ ,  $10^5 = 100000 \equiv 30 \pmod{65}$ ,

**17**  $30^{25}$  means multiply  $30 \times 30 \times 30 \cdots \times 30$ . (That's twenty-five 30's multiplied together.) We can do this in five groups of 5 30's:

$$\begin{aligned}
 30^{25} &= (30 \times 30 \times 30 \times 30 \times 30) \times (30 \times 30 \times 30 \times 30 \times 30) \times (30 \times 30 \times 30 \times 30 \times 30) \\
 &\quad \times (30 \times 30 \times 30 \times 30 \times 30) \times (30 \times 30 \times 30 \times 30 \times 30) \\
 &= 30^5 \times 30^5 \times 30^5 \times 30^5 \times 30^5 \\
 &= (30^5)^5
 \end{aligned}$$

**18** 71

**19** 17